6.S: Gauss's Law (Summary)

Key Terms

- **area vector**: vector with magnitude equal to the area of a surface and direction perpendicular to the surface

- **cylindrical symmetry**: system only varies with distance from the axis, not direction

- **electric flux**: dot product of the electric field and the area through which it is passing

- **flux**: quantity of something passing through a given area

- **free electrons**: also called conduction electrons, these are the electrons in a conductor that are not bound to any particular atom, and hence are free to move around

- **Gaussian surface**: any enclosed (usually imaginary) surface

- **planar symmetry**: system only varies with distance from a plane

- **spherical symmetry**: system only varies with the distance from the origin, not in direction
Key Equations

Definition of electric flux, for uniform electric field
\[
\Phi = \vec{E} \cdot \vec{A} \cos \theta
\]

Electric flux through an open surface
\[
\Phi = \int_S \vec{E} \cdot \hat{n} \, dA = \int_S \vec{E} \cdot d\vec{A}
\]

Electric flux through a closed surface
\[
\Phi = \oint_S \vec{E} \cdot \hat{n} \, dA = \oint_S \vec{E} \cdot d\vec{A}
\]

Gauss’s law
\[
\Phi = \oint_S \vec{E} \cdot \hat{n} \, dA = \frac{q_{\text{enc}}}{\varepsilon_0}
\]

Gauss’s Law for systems with symmetry
\[
\Phi = \oint_S \vec{E} \cdot \hat{n} \, dA = E \oint_S dA = EA = \frac{q_{\text{enc}}}{\varepsilon_0}
\]

The magnitude of the electric field just outside the surface of a conductor
\[
E = \frac{\sigma}{\varepsilon_0}
\]

Summary

6.2 Electric Flux

- The electric flux through a surface is proportional to the number of field lines crossing that surface. Note that this means the magnitude is proportional to the portion of the field perpendicular to the area.
- The electric flux is obtained by evaluating the surface integral
\[
\Phi = \int_S \vec{E} \cdot \hat{n} \, dA = \int_S \vec{E} \cdot d\vec{A}
\]

where the notation used here is for a closed surface $S$.

6.3 Explaining Gauss’s Law

- Gauss’s law relates the electric flux through a closed surface to the net charge within that surface,
\[
\Phi = \oint_S \vec{E} \cdot \hat{n} \, dA = \frac{q_{\text{enc}}}{\varepsilon_0}
\]

- where $q_{\text{enc}}$ is the total charge inside the Gaussian surface $S$.
- All surfaces that include the same amount of charge have the same number of field lines crossing it, regardless of the shape or size of the surface, as long as the surfaces enclose the same amount of charge.
6.4 Applying Gauss’s Law

- For a charge distribution with certain spatial symmetries (spherical, cylindrical, and planar), we can find a Gaussian surface over which \( \vec{E} \cdot \hat{n} = E \), where \( E \) is constant over the surface. The electric field is then determined with Gauss’s law.
- For spherical symmetry, the Gaussian surface is also a sphere, and Gauss’s law simplifies to \( 4\pi r^2E = \frac{q_{\text{enc}}}{\varepsilon_0} \).
- For cylindrical symmetry, we use a cylindrical Gaussian surface, and find that Gauss’s law simplifies to \( 2\pi rLE = \frac{q_{\text{enc}}}{\varepsilon_0} \).
- For planar symmetry, a convenient Gaussian surface is a box penetrating the plane, with two faces parallel to the plane and the remainder perpendicular, resulting in Gauss’s law being \( 2AE = \frac{q_{\text{enc}}}{\varepsilon_0} \).

6.5 Conductors in Electrostatic Equilibrium

- The electric field inside a conductor vanishes.
- Any excess charge placed on a conductor resides entirely on the surface of the conductor.
- The electric field is perpendicular to the surface of a conductor everywhere on that surface.
- The magnitude of the electric field just above the surface of a conductor is given by \( E = \frac{\sigma}{\varepsilon_0} \).

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