Topology and general relativity

Topology is, roughly speaking, the study of those properties of a space that don't change under continuous deformations. The standard joke is that "a topologist is someone who can't tell a donut from a coffee cup." If you have a donut-shaped object made of soft clay, you can deform it into a coffee cup-shaped object without cutting or tearing. A donut is not, on the other hand, topologically the same as a baseball -- to make a baseball-shaped object into a donut-shaped one, you have to cut a hole in the middle.

The field equations of general relativity determine the geometry of spacetime in terms of the matter content. They do not, in general, determine the topology. The two aren't completely independent; choices of geometry and topology must be compatible, and this places some restrictions on possible spacetimes. As a simple example, the average curvature of a two-dimensional manifold with the topology of torus (that is, the surface of a donut) must be zero, while the the average curvature of a two-dimensional sphere must be positive. But the restrictions are weak, and many topologies are consistent with the same geometry. In an earlier section of this glossary, for example, I described a geometrically flat torus. But there are also nonflat geometries on a torus (picture a donut again), and there are other two-dimensional spaces -- the plane, for instance -- with zero curvature.

One of the interesting problems in modern cosmology is that of determining the topology of space in the real Universe. An interesting discussion of the problem is here. The September 1998 issue of Classical and Quantum Gravity has the proceedings of a conference on the topology of the Universe.

Another interesting question is whether quantum fluctuations can cause the topology of space to change in time. John Wheeler has proposed a picture of "spacetime foam," in which the topology of the Universe at the smallest scales is undergoing complicated, random fluctuations. Whether this picture is correct or not remains an open question.

For a nice nontechnical introduction to topology, see Jeff Weeks' book, The Shape of Space. For a little more detail, a good
elementary introduction is Klaus Janich’s book, *Topology*.

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**Contributors and Attributions**

- Steve Carlip (Physics, UC Davis)