3.4: Nuclear mass formula

There is more structure in Figure 4.3.1 than just a simple linear dependence on \( A \). A naive analysis suggests that the following terms should play a role:

1. **Bulk energy**: This is the term studied above, and saturation implies that the energy is proportional to 
   \( B_{\text{bulk}} = \alpha A \).

2. **Surface energy**: Nucleons at the surface of the nuclear sphere have less neighbors, and should feel less attraction. Since the surface area goes with \( R^2 \), we find 
   \( B_{\text{surface}} = -\beta A \).

3. **Pauli or symmetry energy**: nucleons are fermions (will be discussed later). That means that they cannot occupy the same states, thus reducing the binding. This is found to be proportional to 
   \( B_{\text{symm}} = -\gamma (N/2-Z/2)^2 A^{-1} \).

4. **Coulomb energy**: protons are charges and they repel. The average distance between is related to the radius of the nucleus, the number of interaction is roughly \( Z^2 \) (or \( Z(Z-1) \)). We have to include the term 
   \( B_{\text{Coul}} = -\epsilon Z^2 A^{-1/3} \).

Figure \( \PageIndex{1} \): Illustration of the terms of the semi-empirical mass formula in the liquid drop model of the atomic nucleus. (CC BY-SA; Daniel FR).

Taking all this together we fit the formula

\[
B(A,Z) = \alpha A - \beta A^{2/3} - \gamma (A/2-Z)^2 A^{-1} - \epsilon Z^2 A^{-1/3}
\]

\[\label{eq:mass1}\]

to all know nuclear binding energies with \( A \geq 16 \) (the formula is not so good for light nuclei). The fit results are given in
Table \( \PageIndex{1} \).

Table \( \PageIndex{1} \): Fit of masses to Equation \ref{eq:mass1}.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>15.36 MeV</td>
</tr>
<tr>
<td>( \beta )</td>
<td>16.32 MeV</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>90.45 MeV</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>0.6928 MeV</td>
</tr>
</tbody>
</table>

Figure \( \PageIndex{2} \): Difference between fitted binding energies and experimental values (color), as a function of \( N \) and \( Z \).

In Table \( \PageIndex{1} \) we show how well this fit works. There remains a certain amount of structure, see below, as well as a strong difference between neighbouring nuclei. This is due to the superfluid nature of nuclear material: nucleons of opposite momenta tend to anti-align their spins, thus gaining energy. The solution is to add a pairing term to the binding energy,

\[
B_{\text{pair}} = \begin{cases} 
A^{-1/2} & \text{for $N$ odd, $Z$ odd} \\
- A^{-1/2} & \text{for $N$ even, $Z$ even}
\end{cases}
\]

The results including this term are significantly better, even though all other parameters remain at the same position (Table \( \PageIndex{2} \)). Taking all this together we fit the formula

\[
B(A,Z) = \alpha A - \beta A^{2/3} - \gamma (A/2-Z)^2A^{-1} - \delta B_{\text{pair}}(A,Z)-\epsilon Z^2 A^{-1/3}
\]

\( \label{eq:mass2} \)
Table \(\PageIndex{2}\): Fit of masses to Equation \(\ref{eq:mass2}\).

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
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</thead>
<tbody>
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<tr>
<td>(\beta)</td>
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<td>(\gamma)</td>
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<tr>
<td>(\delta)</td>
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<tr>
<td>(\epsilon)</td>
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</tbody>
</table>

Figure \(\PageIndex{3}\): \(B/A\) versus \(A\), mass formula subtracted.