9.1: Nuclear Structure

Nucleons

We begin our examination of atomic nuclei with a sense of the size scale that we are talking about. It's tempting to think of electrons crowding around a nucleus, making a very compact atom, but if we compare the radius of a proton (which itself is made up of bound states of smaller particles called quarks) to the separation of the electrons from the nucleus, a different picture emerges about the space that exists within an atom. The ratio of Neptune's orbital radius to the radius of the sun is about 6,500. The ratio of a typical electron orbit to the radius of a typical nucleus is close to 10 times this much. For example, a Bohr orbit for the hydrogen atom is about one-half angstrom ($10^{-10} \text{m}$), while a nucleus is on the order of a femtometer ($10^{-15} \text{m}$). As small as atomic orbits seemed to be, nuclear scales are significantly smaller.

There are two particles that show up in atomic nuclei (both of them composed of quarks) – protons and neutrons. Collectively they are known as nucleons. We give them this generic designation because aside from the differences in their electric charge, they are virtually identical – very close in mass (we will treat them as having equal masses), and react in the same manner to nuclear forces. These particles are both fermions, so the exclusion principle will apply to them (but of course protons cannot exclude neutrons or vice-versa, because despite their similarities, they are clearly distinguishable from each other).

As fermions, both of these particles have spin. One might ask how one can experimentally measure the spin of a neutron. The usual way is to put a beam through a Stern-Gerlach apparatus, and the magnetic moment of the particles causes the deflection needed to measure the spin. But as the neutron has no charge, it must have no magnetic moment, right? Wrong! The neutron is neutrally-charged overall, but its constituent pieces (the quarks) are charged – the total charge of the quarks just sums to zero. The neutron's magnetic moment is roughly two-thirds of that of the proton, and points in in the opposite direction relative to its spin (i.e. its magnetic moment points in the same direction relative to its spin as if it were a negatively-charged particle).
Note that the magnetic moment of a particle with spin is given by:

\[
\vec{\mu} = g \frac{q}{2m} \vec{S}
\]

With masses close to 2000 times greater than the electron, neither the neutron nor the proton has a magnetic moment close to that of the electron. This is why the nucleus does not play much of a role in the deflection of beams of atoms passing through a Stern-Gerlach apparatus.

The proton and neutron are so close to each other that on occasion they turn into one another. A neutron can undergo a process called beta decay, whereby it transforms into a proton plus an electron (the electric charge remains conserved), plus another particle called an antineutrino. This last particle is a neutrally-charged, nearly massless spin-\(\frac{1}{2}\) fermion, and it assures that all of the conserved quantities (energy, momentum, angular momentum) come out right. It is very elusive to measure, as it interacts only very weakly with anything. This particle is designated with the symbol \(\overline{\nu}_e\) (the "e" in the subscript indicates that this antineutrino is related to the electron, as there are actually more types of neutrinos).

The reverse process of inverse beta decay – a proton turning into a neutron plus an antielectron (called a positron) and neutrino (symbolized by \((\nu_e)\)) also occurs. Note that the word "beta" used here is a holdover from the earliest days of atomic physics – a "beta particle" is an electron or a positron, and it designated with the symbol \(\beta^-\) (electron) or \(\beta^+\) (positron).

### Modeling Nuclei

Nucleons attract each other through the strong nuclear force that attracts the quarks within them toward each other. This force acts independent of other forces, such as the electromagnetic repulsion of the protons, and it is very different from the other two fundamental forces we are familiar with (E&M and gravity). The details of the strong nuclear force are, like quarks, in the purview of particle physics, but there is one basic feature of it that is important to know – it is a short-range force. Gravity and E&M have infinite ranges, and get weaker as the separation grows, but the strong nuclear force is just the opposite – as quarks get pulled apart, the force grows stronger, until a point when the force very suddenly drops to zero.

Whether the nucleons involved are both protons, both neutrons, or one of each, the strong nuclear force is attractive. While the strong nuclear force is the strongest of all the fundamental forces, this strength is manifested mainly in attracting quarks to each other within nucleons. When nucleons get close to each other, the quarks within one nucleon do attract quarks within the other nucleon, but the strength of this force is greatly diminished compared to the force between the quarks internal to the nucleon. If they get too close then they very strongly repel each other, as if they have hard spherical cores like billiard balls.

With this description of the force between nucleons, we can create a simple 1-dimensional quantum-mechanical model. We seek a potential that is attractive at short distances (a well), with an infinite barrier at a zero separation (hard-sphere repulsion), and a force that ends abruptly at some separation (finite wall). In short, a potential well that looks something like this:

Figure 9.1.1 – Two Nucleon Potential Well Model
We know that in one dimension the infinite square well has infinitely-many bound states, and a finite square well has at least one bound state, but surprisingly this well is not guaranteed to have any bound states whatsoever! For a given nucleon mass and maximum range of the force (\(\langle R \rangle\)), there is a minimum depth (\(\langle V_o \rangle\)) that this well can have that will support a bound state.

Unfortunately we find experimentally that this model is just a bit too simple. It turns out that the strong nuclear force includes in its calculation the spins of the two particles involved. The force of attraction is greater (i.e. the well is deeper) when the spins of the two particles are aligned (i.e. both are spin up or both are spin down). The difference is great enough that if the spins are not aligned, then the well is not deep enough to allow for a bound state to exist. This spells doom for the two-neutron and two-proton bound states, because such pairs cannot coexist within the same well with all the same quantum numbers, due to the exclusion principle.

So we find that only one proton and one neutron can form a bound state, provided their spins are aligned (and the combined particle has a spin of 1 as a result). Such a particle is called a deuteron. This particle as the same charge as a proton, and double the mass and can be part of (for example) a hydrogen atom (called deuterium, or heavy hydrogen). As the same atoms can come with different numbers of neutrons, we call the different varieties isotopes, and label them with the number of nucleons, such as "uranium-238" or "carbon-14." These are often written with the atomic symbol and two numbers that indicate the number of protons (as a reminder, in case you don't have the periodic table memorized), and the total number of nucleons. For example, carbon-14 is written as "\(^{14}\_{\;6}C\)."

All isotopes of the same atom behave the same way chemically (they have the same number of protons and electrons, giving them the same shell structure and valences), so they can be bound into molecules in the same way. A water molecule built from deuterium is known as heavy water.

One might wonder how more nucleons can be bound together in a nucleus if we can't even put two protons together. Well, adding another neutron into a three-particle bound state (called a triton, which is a nucleus for hydrogen-3, also known as tritium) is possible. The second neutron must have the opposite spin of the first, and this increases the potential energy, but
now there are *three* negative-potential-energy interactions (each nucleon pulling on each other), rather than just one, and this makes the well deep enough to keep the spin-spin interaction from pushing one of the neutrons away. There is a fundamental difference between tritium and deuterium, however, which is our next topic for discussion.

**Stability**

When we talk about particles in bound states, the model shown in Figure 9.1.1 assumes that when the separation becomes great enough that the strong nuclear force goes away, the nucleons are simply free of each other (i.e. by our usual convention, the potential is zero). While this is pretty much true of the proton and neutron in the deuteron, it is not this simple when more nucleons are added into the nucleus. In the example of tritium, the opposite spins of the two neutrons result in a repulsive interaction that, while it gets weaker with separation, is still present at distances beyond the influence of the strong nuclear force. This means that in such cases, the top of the well at what we called \(r=R\) in Figure 9.1.1 is not actually zero potential energy – it is the positive potential energy that comes from the repulsive force. The force gets weaker as the separation grows, so the potential energy drops at larger values of \(r\). A simplified blocked version of this characteristic of the potential is shown in Figure 9.1.2. The key point here is that if the bound state is high enough energy, the barrier will allow for tunneling. If the bound state energy is deeper into the well, then tunneling is not possible. Nuclei that satisfy these conditions are referred to as *unstable* and *stable*, respectively. The process of the nucleus losing a nucleon to tunneling is called *radioactive decay*.

![Figure 9.1.2 – Stable and Unstable Bound States](image)

Notice that the figure indicates that the particle *eventually* tunnels out. What does this mean, exactly? What we know about tunneling is that a particle that strikes a barrier has some probability of tunneling or being reflected. Well, if we think about this particle classically, it is essentially bouncing back-and-forth between the infinite wall and the finite one. Every time it strikes the finite wall, it has some chance of tunneling through. With every individual event there is an equal probability of tunneling out, but the probability of tunneling out after \(N\) tries is better. We can (in principle) compute how many attempts are needed for the particle to have a 1/2 probability of tunneling out, and translate this into a time period. This time period is called the *half life* of this nucleus. Notice that if there are \(N\) such nuclei and each of them has a one-half probability to
decay after one half-life, then we expect only \( \frac{N}{2} \) of the original nuclei to remain.

In the case of tritium, the bound state energy is in fact high enough (due to the spin-spin interaction) that it is not stable, while the lack of a spin-spin interaction for the deuteron makes it stable. As a rule, stable nuclei are the ones that we encounter in nature, and those that are unstable are ones that we construct ourselves. This is because naturally-occurring nuclei were constructed in the cores of stars, and were then dispersed via supernovae billions of years ago, giving the unstable nuclei plenty of time to decay, leaving behind the stable nuclei. There are some exceptions, however. Uranium-238, for example, is unstable with a half-life of about 4.5 billion years, so even though it was formed very long ago, there is still some around – it is naturally-occurring.

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**Large Nuclei**

As we build a nucleus by adding more nucleons, assuming the nucleons are all within the range of the strong force from each other, the well gets deeper and deeper. But if the nucleons include protons, then they interact with each other through EM repulsion, taking away some of that increased well depth for the protons (and not affecting it for the neutrons). So one might think that the best way to build a stable large nucleus is to only tack-on neutrons. Although this doesn’t change the depth of the well for the nucleons, these particles climb even faster upward in the well, because they fill up energy levels two at a time, thanks to the exclusion principle. Mixing in some protons (which are not identical to neutrons) adds some coulomb potential energy, but also allows more particles into lower energy levels within the well.

There are four main ingredients that decide how the heavier nuclei are formed:

- Adding more nucleons increases the number of strong force interactions, deepening the well, to a point (see below).
- Adding protons increases their potential energy through the electromagnetic interaction.
- Adding protons or neutrons at a given energy level is limited by the exclusion principle.
- When enough nucleons are added, they cannot all stay close enough to each other to interact, so the addition of more nucleons does not decrease the potential energy per particle anymore, making the binding of new nucleons much weaker.

The figure below gives a pictorial representation of what happens as the nucleus is built larger and larger.

**Figure 9.1.3 – Building a Larger Nucleus**
When enough protons have been added, the coulomb "bump" in the bottom of the well gets high enough that more neutrons fit in the well for the same total energy, explaining why larger nuclei are more laden with neutrons than protons.