5.1: Steady-State Energy-Density Model

Most traditional physics textbooks discuss the flow of fluids and the flow of electric charge in electric circuits as completely separate topics. Our goal in this present chapter is to understand both kinds of transport phenomena using the Steady-State Energy Density Model. Historically, different words and symbols have been used for the description of each of these phenomena, making their similarity even more difficult to see. We will generally use conventional notation and vocabulary, because this is the notation you will see and use in your other science courses and in research. While becoming comfortable with the specific notation and vocabulary of each of these different types of steady-state transport phenomena, you need to simultaneously become conscious of the more universal nature of the underlying model and approach. In this way, you can use your understanding in one area to help develop understanding in other less familiar areas.

The principle of conservation of energy applied to fluid phenomena is expressed in a relation historically known as Bernoulli’s equation. When dissipation effects and sources of energy input are included, the term “extended Bernoulli equation” is sometimes used. We will generally use the terms “energy-density equation” and “complete energy-density equation” for both fluid flow and electric circuit phenomena. Changes in the total energy of a small element of fluid include changes in its kinetic energy, its gravitational potential energy, and its pressure. The first two terms should be familiar and don’t present any difficulty in understanding what they mean. The pressure term, however, is more challenging. The energy (actually energy density) represented by the pressure is related to whatever it is that confines the liquid and gives rise to the pressure. For the common occurrence of static incompressible fluids in open containers, the pressure is ultimately due to gravitational forces acting on the fluid.

Relationships of exactly the same form and meaning as used to make sense of fluid phenomena are used to express energy conservation in electric circuits. Instead of pressure, the “fluid” energy density is the “voltage.” This is the same voltage you are familiar with when installing 1.5 or 9 Volt batteries in your electronic gadgets. We will develop and use the steady state energy density model in terms of both real fluid flow and electric circuits. Our goal is to make sense of both kinds of
phenomena using a common model, and thus take advantage of the understanding we have in one domain to help us make sense of the other. Both static-fluid and flowing-fluid phenomena are very common, of course, in living organisms.

**Pressure and Other Energy Densities in Fluids**

Fluids include gases and liquids. The distinguishing feature of fluids is that the individual molecules do not have fixed positions relative to one another. We will introduce several parameters that characterize fluids, some of which apply to solids as well. One parameter that applies only to fluids is pressure, which is where we begin.

What is pressure? You are familiar, of course, with this word from both everyday usage and within a science context. Fundamentally, pressure is an intensive property of fluids, just like temperature or density is an intensive property. By intensive, we mean the property does not scale with the volume of the substance as energy and volume do. While solids are characterized by temperature and density, only fluids (liquids and gases) have a characteristic pressure. For fluids, then, pressure is another state variable. It tells us something about the state of the fluid system. The common use of the term “pressure” in a phrase such as “I can really feel the pressure building up inside me as finals approach; I think I’m about to explode!” somewhat captures the spirit of the meaning of pressure in a fluid. The pressure in a fluid can be changed by external means, and an increase in pressure represents an increase in energy.

Perhaps one of the most useful ways to think of pressure is as an energy density. More precisely, the pressure at a particular point in a fluid is the energy per unit volume that must be transferred from another system into the fluid system in order to create a unit volume of fluid at that point. Now consider a small volume of fluid $(V)$. We can think of this as just a small, macroscopic amount of fluid—much more than a single molecule, but small enough so that all parameters (such as pressure) are constant throughout the volume $(V)$. We focus on the changes of energy $\Delta E$ of this volume of fluid.

![Figure 5.1.1](image)

The energy $E_{\text{total}}$ of the fluid element of volume $(V)$ will consist of the internal energy $(U)$ (thermal plus all bond systems) plus any macroscopic energies. In particular, we will want to include the gravitational potential energy, because we want to consider changes in elevation of the fluid element. We also want to include the kinetic translational energy of the fluid element in order to consider the fascinating phenomena that occur when the speed of fluids change. We will omit in this treatment rotational motions like the vortices that sometimes form when water goes down the drain of your bathtub.
The gravitational potential energy of the small volume of fluid is \( \rho Vgy \), where \( y \) is measured from some convenient reference point. The mass of the small volume of fluid is \( \rho V \), where \( \rho \) is the mass density. The gravitational potential energy density is then

\[
\rho gy = \text{gravitational potential energy density}.
\]

The translational kinetic energy of the small volume \( V \) of fluid is \( \frac{1}{2} \rho Vv^2 \) and the kinetic energy density is

\[
\frac{1}{2} \rho v^2 = \text{kinetic energy density}.
\]

If we focus only on frictionless flow for the time being, we can combine the gravitational potential energy density and kinetic energy density terms with the pressure energy density in a conservation of energy density equation 5.1.1.

\[
\rho g \Delta y + \frac{1}{2} \rho \Delta (v^2) + \Delta P = 0. \tag{5.1.1}
\]

Before proceeding further, we step back and compare the two energy conservation approaches.

**Differences in the Two Basic Energy Models**

The most general and universal approach to getting answers to questions about phenomena in the physical world is to use conservation principles. The most universal of these principles is conservation of energy. However to actually use this very general principle of conservation of energy, we need to cast it in a form that is useful for analyzing particular types of phenomena.

Here is a comparison of the energy-interaction model from Chapters 1 and 2 of Part 1 and the steady-state energy density model for fluids and electricity that we are developing in this chapter. The figure 5.1.2 shows both the similarities and differences in our two energy conservation models.

To use either of these models, we follow these basic steps in carrying out the approach:
1. From the various physical interactions occurring, identify the specific energy (energy density) systems that significantly change in energy during the interaction (as we move from one position to another), along with the observable parameter from which the change in energy (energy-density) can be calculated.

2. Determine useful initial and final states (two useful positions) of the physical system. Which states (positions) we choose is determined by the questions we are asking.

3. Write down an equation expressing conservation of energy, using energy (energy density) diagrams or whatever else is helpful. One way to write this equation is as a sum of the changes in energy (energy-density) of each of the energy (energy-density) systems. If no energy is added or removed from outside systems, i.e., a closed system, the sum is set equal to zero. If there are outside sources of energy, i.e., an open system, then the sum is set equal to those energy sources.