4. Graphical Representations of Waves

Now one of the tricky things about the solution to the wave equation expressed here

\[ y(x, t) - y_0 = A \sin \left( 2 \pi \left( \frac{t}{T} \pm \frac{x}{\lambda} \right) + \phi \right) \]

is that it is a function of both space (the distance along the \(x\)-axis) and a function of time (the value of the time variable, \(t\)).

One way to make visualising the equation easier is to think of either \(x\) or \(t\) as being fixed at some value, so that \(y\) depends on only one variable. This is exactly what we must do in order to graph the function \(y(x, t)\) on a simple 2D graph. We will describe these two graphs for the same example wave below.

**Holding \(t\) constant: Displacement \(y\) vs. Position \(x\)**
This graph shows the displacement of the entire wave at a particular time \( t \), hence the name "Snapshot." The graph serves as an apt visual representation of a transverse wave. Although it does not visually represent the form of a longitudinal wave, the graph is correct for both kinds of wave.

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**Holding \( x \) constant: Displacement \( y \) vs. time \( t \)**

![Graph showing displacement vs. time](image)

This graph plots the motion of a particular piece (fixed \( x \)) of the wave against time. This graph reminds us that this particular piece of the medium is undergoing simple harmonic motion.

To convey all of the information contained in \( y(x,t) \) requires both graphs. Alternatively, almost all of the same information can be conveyed using two displacement vs. position graphs for two different known times, or two displacement vs. time graphs for two different known positions.

**Exercise**

Using the two graphs above, can you find the equation for the wave? You should be able to get numerical values for all the variables except one. Which one can’t you determine? Hint: Which way is the wave travelling (left or right)? You will need this to determine the + or - sign. Which parameters do you get from which graph? Which parameters require both graphs?