5.5: Simple Circuit Analysis Techniques

The examples we just worked through have shown how we can use the steady-state energy density model to calculate various fluid flow or charge flow parameters given sufficient details about the physical situation. Now we extend this analysis to complete “circuits.” There is not any really new physics in what we are about to do, but it is certainly useful to learn some of the electrical engineers’ little tricks for analyzing the kinds of circuits we encounter in our daily lives.

It is useful to combine the complete energy-density equations with conditions on charge or fluid conservation into some practical rules. We will apply the circuit analysis rules arising from that combination to some practical fluid and electrical circuits.

Generalized Continuity Equation

Previously we wrote the Continuity Equation, which is another way of expressing the conservation of fluid volume:

\[ A_1 v_1 = A_2 v_2 = I. \]

This can be extended to include the effects of having multiple pipes joined together in junctions, or branching off in several directions. We do not have to be limited to only a single pipe!
Figure 5.5.1

Current that flows into a junction must be equal to the current that flows out. This is frequently called the junction rule by electrical engineers, but it is a really just a statement of conservation of fluid applicable to all fluid transport phenomena.

Concept of Complete Circuit

(Note: in this section, we will mostly talk about electric circuits and use the symbols for electric circuits. However, the same relationships exit for complete fluid circuits.) Up to this point, we have treated the current as an independent variable, as we looked at drops in potential (or head for fluid systems). However, frequently, we deal with complete circuits. That is, the complete path of the charge flow is topographically equivalent to a circle or complete loop. The current is no longer an independent variable; rather, the various resistances and sources of emf (batteries or generators) will determine the current that exists in the circuit.

Figure 5.5.2
First, some standard notation and use of symbols. It is customary to indicate batteries and resistors as shown in the figure 5.5.2. Also, it is customary to draw the wires connecting the electrical components as straight and usually in either the vertical or horizontal direction.

The figure 5.5.2 illustrates a section of a circuit that contains both a battery that increases the energy density of the electrical charge that flows through it by an amount \(+\varepsilon\), and a separate resistive section that decreases the potential of the electric charge that flows through it by an amount \(IR\). Thus:

\[ \Delta V_{1 \text{ to } 2} = +\varepsilon - IR \]

In going from point 1 to point 2, the battery increases the electrical potential by \(+\varepsilon\) while some energy is transferred to thermal systems by the resistor. Whether the potential at point 2 is higher or lower than at point 1 depends on the relative amounts of increase due to the battery and decrease due to the resistor.

Now imagine what happens if the charge is transported around a complete loop? Like our previous example, this circuit includes a battery, and a resistor, but the current continues back to its original starting point. Because point 1 and 2 are connected by what we are modeling as a zero resistance wire, \(\Delta V_{2 \text{ to } 1} = 0\). That is, point 1 and 2 must be at the same potential, because there is nothing separating them but a zero-resistance wire. Electrically, they are the same point. So,

\[ \Delta V_{1 \text{ to } 2} = \Delta V_{1 \text{ to } 1} = +\varepsilon - IR = 0, \]

or

\[ \varepsilon = IR \]

If there is more than one battery and/or resistor in a complete circuit, then energy density conservation says that for any loop that comes back on itself, the sum of the sources of emf and the voltage drops across the resistors must sum to zero. We have to get back to the same potential.

\[ \sum \varepsilon - \sum (IR) = 0 \]
In words, this equation (5.5.6) states that for any complete loop of circuit (no matter how complicated the path appears, and how many batteries and resistors are in the loop), the total increase in potential caused by the batteries must equal the total IR losses caused by all resistors in the loop. This is known as the loop rule in electricity, but it is a formal statement of energy density conservation applicable to any fluid transport phenomena.

Method of Equivalent Reduction

In analyzing circuits, the most straightforward method is to apply both the junction and loop rules and translate them into algebraic equations to solve for any unknown quantities. In complicated circuits there will typically be multiple complete loops. By simply writing down the loop rule for enough loops, you can eventually get sufficient number of equations to solve for the number of unknowns.

For many simple circuits of practical significance, we can reduce sets of circuit elements (batteries and resistors) into simpler equivalent circuit elements. We will consider only circuits that can be solved using this equivalent reduction method.

Resistors in series or parallel are equivalent to a single resistor in terms of the currents and potential changes in the remainder of the circuit. These equivalent resistance values may be found by applying the junction and loop rules.

For resistors in series, the equivalent resistance is just the algebraic sum of the individual resistance:

\[ R_{\text{series}} = R_1 + R_2. \]

Figure 5.5.4
Example: Calculating Resistance: Analysis of a Series Circuit

Suppose the voltage output of the battery in Figure is \(12.0\,\text{V}\), and the resistances are \(R_1=1.00\,\Omega\), \(R_2=6.00\,\Omega\), and \(R_3=13.0\,\Omega\). What is the total resistance?

\[
R_{\text{S}} = R_1 + R_2 + R_3
\]

\[
= 1.00\,\Omega + 6.00\,\Omega + 13.0\,\Omega
\]

\[
= 20.0\,\Omega.
\]

Strategy and Solution

The total resistance is simply the sum of the individual resistances, as given by this equation:

\[
R_{\text{S}} = R_1 + R_2 + R_3
\]

\[
= 1.00\,\Omega + 6.00\,\Omega + 13.0\,\Omega
\]

\[
= 20.0\,\Omega.
\]
For resistors in parallel, the reciprocal of the equivalent resistance is just the sum of the individual reciprocal resistances:

\[
\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2}
\]

*Figure 5.5.5*

If sources of emf (such as batteries) are hooked up in series, their \( \varepsilon \)'s add algebraically. The sign of the \( \varepsilon \) of a battery is positive if the current enters the negative terminal and exits the positive terminal.

\[
\varepsilon_{\text{series}} = \varepsilon_1 + \varepsilon_2
\]

*Figure 5.5.6*

Note: Adding batteries in parallel is not normally done, because the equivalent voltage depends on the internal resistance as well as the emfs of each one separately.
Example: Calculating Resistance: Analysis of a Parallel Circuit

Let the voltage output of the battery and resistances in the parallel connection in Figure be the same as the previously considered series connection: \( V = 12.0 \text{ V}, R_1 = 1.00 \Omega, R_2 = 6.00 \Omega \), and \( R_3 = 13.0 \Omega \). What is the total resistance?

Strategy and Solution

The total resistance for a parallel combination of resistors is found using the equation below. Entering known values gives

\[
\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1.00 \Omega} + \frac{1}{6.00 \Omega} + \frac{1}{13.0 \Omega}. 
\]

Thus,

\[
\frac{1}{R_p} = \frac{1.00}{\Omega} + \frac{0.1667}{\Omega} + \frac{0.07692}{\Omega} = \frac{1.2436}{\Omega}. 
\]

(Note that in these calculations, each intermediate answer is shown with an extra digit.)

We must invert this to find the total resistance \( R_p \). This yields

\[
R_p = \frac{1}{1.2436} \Omega = 0.8041 \Omega. 
\]

The total resistance with the correct number of significant digits is \( R_p = 0.804 \Omega \).

Contributors

• Authors of Phys7B (UC Davis Physics Department)