9.3.2: Lenses and Ray Tracing

In this section we will learn how to use focal points to determine how lenses bend light rays. For each of our lenses there are three rays which are easy to find if we have knowledge of just the focal point and the position of the object – we call these special rays principal rays. We will show how to find the principal rays for both a converging and diverging lens.

Let us start with some general comments that apply to both converging and diverging lenses. In a lens problem we start with an object, a real thing that reflects real light. Typically we represent this object as an arrow, so that we can tell by looking at the final image if the object is inverted by the lens.

We have discussed what the focal point is for a lens; for a symmetric lens we have two focal points on either side. For a converging lens, rays parallel to the optical axis incident to the right side of the lens are focused on the left, parallel rays from the left are focused on the right.

The focal length \( f \) of a lens tells us how far from the lens the focal points are. The magnitude of the focal length is the distance from the lens to each focal point, while the sign tells us if the lens is converging or diverging. For a converging lens we take \( f > 0 \), while for a diverging lens we take \( f < 0 \). A typical diagram of a lens and its characteristics (i.e. focal points and optical axis) are shown below.
As we can see from the picture above, the whole lens is typically omitted from visual diagrams of lenses. We are making the approximation that the lens is thin and that refraction (bending) only occurs once at the center of the lens. To make this approximation clear, we represent the lens with a vertical line and the outward and inward arrows to tell us if the lens is converging or diverging, respectively.

How could we find the image by the object formed below? For the moment let us just ask about the tip of the arrow. We know that to find the image of a point we must look at multiple rays from that point and see where they go. We start by drawing many different rays that come off the object. Each of these rays end up being refracted. Out of the incident rays drawn, there are three rays (labelled 1, 2, and 3 below) for which we can draw the refracted ray without doing any math.

Ray 1 is parallel with the optic axis; we know that rays parallel to the optical axis incident on the lens are refracted through the focal point on the other side of the lens. We have also discussed how the lens is thin, so we can approximate ray 2, which passes through the center, as passing through a thin sheet of glass; it isn't refracted appreciably.

The conclusions we make above can be displayed visually:

The ray labelled 3 takes a little more thought. First, recall from Snell's Law that refraction occurs symmetrically in either direction (it does not matter which medium takes the label "1" or "2"; they're reversible). As a consequence, we see that light that enters the lens through a focal point is refracted in a direction parallel to the optical axis.

The diagram below shows this conclusion in ray 3 explicitly:
Notice that all these rays cross at a particular location. That is where the arrow tip will appear to be to any observer off to the far right of the lens. The light rays traveling past this point look just as if the real arrow tip had been placed here. Because the lens designer (we presume) made the lens precisely, all the other light rays should be focused just like the first three we drew. Drawing in all the light rays the picture we get is:

Notice that, from this analysis, we only see where the image of the tip of the arrow is going to be. Because we don’t know where the image of the base will be, we have simply drawn question marks at the bottom of the arrow. Points on the optical axis are slightly trickier to trace rays on because all of the principal rays presented there pass through the center of the lens. We will come back to the issue of the optical axis after discussing diverging lenses. Because the light rays actually cross in this example we would call this a real image.

If we are only interested in locating the image, then we only need to find out where the three principal rays intersect. The three principal rays are:

1. The ray that is going into the lens parallel to the optical axis; this ray gets bent to go through the focal point.
2. The ray that passes through the center; this ray does not bend.
3. The ray that passes through the focal point into the lens; this ray is refracted to be parallel with the optical axis.

In typical optics problems we will only draw the principal rays, although it is important to realize that all the light rays that pass through the lens and are responsible for forming the image.

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Our procedure for diverging lenses is similar to the one presented above for converging lenses. First, we must draw the three principal rays. The first principal ray, ray 1, comes off the object parallel to the optical axis, and gets bent so that the ray appears to be coming from the focal point, as shown below. A dotted ray is drawn to show where the light ray appears to come from. We also include ray 2, which passes through the center, unrefracted.
The third principal ray is, again, obtained by considering the reversibility of refraction (and the symmetry of the lens). We know that any light that travels parallel to the optical axis gets bent so that it appears to come from the focal point. Thinking about the light travelling the other way, we can see that light from the arrow aimed towards the focal point on the right side is refracted by the lens to be parallel to the optical axis. We show this below, where we note that the dotted line on the right indicates where the light ray would have gone had the lens not been there.

![Diagram showing the third principal ray](image)

Normally we do not draw where the light ray would have gone; it is only useful for showing you where to draw the line. The dotted line on the left, that shows where the light appears to have come from, is incredibly important however; a person viewing the light from the right of the lens would think that this ray travelled parallel to the optical axis.

Let's put all three rays together:

![Combined diagram](image)

We note that unlike our previous example, the light rays (solid lines) do not actually cross anywhere. However, an observer on the right of our lens would think that the light is coming from the point where the dashed lines intersect. Because the image is located at the intersection of imaginary rays and not real light rays, we call this a virtual image.

![Virtual image diagram](image)

To make it slightly clearer why the image seen by a person on the right is the same as a small arrow where the dotted lines cross, it is useful to remember what an image is. We could ask “if there were no lens, what size object would we need, and where would we need to place it so that the light that reaches us appears exactly the same?” In the above examples, we show how to find this information.
In both examples, you can replace the lens and the original arrow with an arrow at the point where the rays intersect. We see that these configurations appear exactly the same to any observer on the far right. If you are standing on the right, the only information your eyes have is the light rays that enter them, and you have no way of distinguishing these two situations. We can see that before reaching the lens the light rays appear vastly different, but that does not affect an observer to the right of the lens.

We finish this section by summarizing how to find the three principal rays for a diverging lens:

1. The ray that travels parallel to the optical axis refracts so that it appears to be coming from the focal point on the object’s side of the lens.
2. The ray that travels through the center is not deflected.
3. The ray that would pass through the focal point on the far side of the lens refracts to become parallel to the optical axis.

So far we have found the image location for only the tips of the arrows presented. Trying to find the image location for the base of the arrow leads to a problem for the examples considered so far because the base of the arrow has been on the optical axis. For both converging and diverging lenses the optical axis poses an issue. The reason is that the ray that goes through the center of the lens also goes through both the focal points. Our method for constructing the three principal rays fails for any point on the optical axis as all three principal rays are the same.

What we do know, however, is that points on the optical axis have images on the optical axis. We know this because the one principal ray we can draw for a point on the optical axis is always on the optical axis, and we know the image is where the rays that pass through the axis meet up (or appear to come from). Since we have one ray that is always on the optical axis, the image must also be on the optical axis.

So how do we locate where along the optical axis the image forms? One way of finding it is to draw a point that is very close to the optical axis but not quite on it, and perform the ray tracing for that point instead. Another way uses information we will introduce later, the thin lens equation. Whichever way you choose to do it, the result is that the image distance for the base of the arrow is the same as the image distance for the tip of the arrow (provided the tip of the original arrow is directly above the base). From here, we shall draw our images of arrows down (or up) from the tip to the optical axis. If the base of your object is not on the optical axis, a separate ray tracing for the base is necessary to complete the entire image.

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