9.3.3: The Thin Lens Equation

Limits of Ray Tracing

So far, we've been finding images for a given object and lens combination. We can think of other questions that will be of interest to us later, such as “where would we have to place an object to form the image in a particular location?” or “given this object, and that I want the image to be at this location, what is the focal length of the lens I should use?”. Solving all of our lens problems, no matter how they're phrased, comes down to relating the object, image, and focal length of the lens to one another.

We have already learned how to do ray tracings. Although we have presented ray tracings as a way of going from an object to an image, with some care you can answer any of the questions posed above. The drawback of using a ray tracing is that if we want an accurate answer we must make sure that all our lines are carefully measured and that we setup the problem to scale. While a rough ray-tracing is often useful for figuring out what sort of answers we should expect, there is an easier way of getting the precise location of an image: using the thin lens equation.

Defining Object and Image Distance

We introduce \( o \) as the distance between the object and the lens, and \( i \) as the image distance. The location of the image along the optical axis \( i \) depends only on how far the object is from the lens \( o \) and the focal length of the lens \( f \). It does not, for example, depend on the height of the image.
The magnitude of \(\textit{i}\) is the distance between the lens and the image, but \(\textit{i}\) can be either positive or negative. Here, we choose a convention where \(\textit{i} > 0\) for a real image, and \(\textit{i} < 0\) for a virtual image. Note that these are definitions for the sign of \(\textit{i}\), not the definition of if an object is real or virtual. We recall from Lenses and Ray Tracing that the focal length of a lens is positive for converging lenses and negative for diverging lenses. The object distance \(\textit{o}\) is always positive. These ideas are summarized in the figure above and table below.

<table>
<thead>
<tr>
<th>Relevant Sign</th>
<th>Property</th>
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<tbody>
<tr>
<td>(\textit{i} &lt; 0)</td>
<td>Image is virtual</td>
</tr>
<tr>
<td>(\textit{i} &gt; 0)</td>
<td>Images is real</td>
</tr>
<tr>
<td>(\textit{f} &gt; 0)</td>
<td>Converging lens</td>
</tr>
<tr>
<td>(\textit{f} &lt; 0)</td>
<td>Diverging lens</td>
</tr>
<tr>
<td>(\textit{o} &gt; 0)</td>
<td>(Almost) always</td>
</tr>
</tbody>
</table>

The Thin Lens Equation

These three quantities \(\textit{o}\), \(\textit{i}\), and \(\textit{f}\) are related by the thin lens equation

\[
\frac{1}{\textit{o}} + \frac{1}{\textit{i}} = \frac{1}{\textit{f}}
\]

Looking at our previous ray tracings it is apparent that the image and the object do not have to be the same size. This leads us to define the magnification \(\textit{m}\). We define \(\textit{m}\) as the ratio of the height of the image to the height of the object. Thus, if the magnification is \(\textit{m} = 2\) it means that the image is twice as high as the object. We can relate the magnification \(\textit{m}\) to
the object and image distances $i$ and $o$ via

\[ m = \frac{i}{o} \]

Notice that the magnification can be negative. If the image is real (so that $i > 0$) then $m < 0$, meaning that the image would be upside-down. A virtual image has $i < 0$, so $(m)$ is positive, telling us that the image is upright. The advantage to using these particular conventions (rather than conventions based on which side of the lens we are discussing) is that we can use the exact same conventions when discussing curved mirrors with focal points.

(Both of the equations presented above can be derived from what you already know about lenses, but we don't expect you to know the derivation. Still, it is provided in the summary for any interested readers.)

Exercise

If an image is bigger in size than the original object, what does this tell us about the magnification? Does it matter if the image is upright or inverted?

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