Noether's Theorem for Energy

Now we're ready for our first full-fledged example of Noether's theorem. Conservation of energy is a law of physics, and Noether's theorem says that the laws of physics come from symmetry. Specifically, Noether's theorem says that every symmetry implies a conservation law. Conservation of energy comes from a symmetry that we haven't even discussed yet, but one that is simple and intuitively appealing: as time goes by, the universe doesn't change the way it works. We'll call this time symmetry.
b / Emmy Noether (1882-1935). The daughter of a prominent German mathematician, she did not show any early precocity at mathematics --- as a teenager she was more interested in music and dancing. She received her doctorate in 1907 and rapidly built a world-wide reputation, but the University of Göttingen refused to let her teach, and her colleague Hilbert had to advertise her courses in the university's catalog under his own name. A long controversy ensued, with her opponents asking what the country's soldiers would think when they returned home and were expected to learn at the feet of a woman. Allowing her on the faculty would also mean letting her vote in the academic senate. Said Hilbert, “I do not see that the sex of the candidate is against her admission as a privatdozent [instructor]. After all, the university senate is not a bathhouse.” She was finally admitted to the faculty in 1919. A Jew, Noether fled Germany in 1933 and joined the faculty at Bryn Mawr in the U.S.

We have strong evidence for time symmetry, because when we see a distant galaxy through a telescope, we're seeing light that has taken billions of years to get here. A telescope, then, is like a time machine. For all we know, alien astronomers with advanced technology may be observing our planet right now, but if so, they're seeing it not as it is now but as it was in the distant past, perhaps in the age of the dinosaurs, or before life even evolved here. As we observe a particularly distant, and therefore ancient, supernova, we see that its explosion plays out in exactly the same way as those that are closer, and therefore more recent.

Now suppose physics really does change from year to year, like politics, pop music, and hemlines. Imagine, for example, that the “constant” $G$ in Newton's law of gravity isn't quite so constant. One day you might wake up and find that you've lost a lot of weight without dieting or exercise, simply because gravity has gotten weaker since the day before.

If you know about such changes in $G$ over time, it's the ultimate insider information. You can use it to get as rich as Croesus, or even Bill Gates. On a day when $G$ is low, you pay for the energy needed to lift a large mass up high. Then, on a day when gravity is stronger, you lower the mass back down, extracting its gravitational energy. The key is that the energy you get back
out is greater than what you originally had to put in. You can run the cycle over and over again, always raising the weight when gravity is weak, and lowering it when gravity is strong. Each time, you make a profit in energy. Everyone else thinks energy is conserved, but your secret technique allows you to keep on increasing and increasing the amount of energy in the universe (and the amount of money in your bank account).

The scheme can be made to work if anything about physics changes over time, not just gravity. For instance, suppose that the mass of an electron had one value today, and a slightly different value tomorrow. Electrons are one of the basic particles from which atoms are built, so on a day when the mass of electrons is low, every physical object has a slightly lower mass. In problem 14 on page 37, you'll work out a way that this could be used to manufacture energy out of nowhere.

Sorry, but it won't work. Experiments show that $G$ doesn't change measurably over time, nor does there seem to be any time variation in any of the other rules by which the universe works. If archaeologists find a copy of this book thousands of years from now, they'll be able to reproduce all the experiments you're doing in this course.

I've probably convinced you that if time symmetry was violated, then conservation of energy wouldn't hold. But does it work the other way around? If time symmetry is valid, must there be a law of conservation of energy? Logically, that's a different question. We may be able to prove that if A is false, then B must be false, but that doesn't mean that if A is true, B must be true as well. For instance, if you're not a criminal, then you're presumably not in jail, but just because someone is a criminal, that doesn't mean he is in jail --- some criminals never get caught.

Noether's theorem does work the other way around as well: if physics has a certain symmetry, then there must be a certain corresponding conservation law. This is a stronger statement. The full-strength version of Noether's theorem can't be proved without a model of light and matter more detailed than the one currently at our disposal.

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**Contributors and Attributions**

- Benjamin Crowell, *Conceptual Physics*