7.2: Applications of Momentum Conservation

As an example of the use of the application of momentum conservation, consider the collision of two automobiles. The total momentum of the system of two autos is the sum of the individual momenta before the collision. If the forces exerted by the road in the horizontal direction are small compared to the forces exerted by the autos on each other (usually a very good approximation), then the momentum of the system of two autos is conserved. That is, it is the same immediately after the collision as it was before. Why? Because the forces the two cars exert on each other are internal forces and don’t contribute an external impulse. Note that this would not be true if a car hits the proverbial brick wall, since we would typically take the system to be the car, and the brick wall would exert a net external impulse to the system. The diagram(figure 7.2.1) shows a not quite head-on collision between car (a) moving to the right with a faster and/or heavier car moving to the left:

![Figure 7.2.1](image)

Regardless of what happens during the collision, as long as the road exerts a negligible impulse during the collision (compared to the impulses exerted by the colliding autos on each other), the total momentum of the two autos immediately before the
collision equals the total momentum immediately after the collision. Note that in the example shown in the figure, the total momentum before and after the collision is shown in the right part of the diagram, and the vectors are equal. They are equal in spite of the fact that after the collision, the autos bounce off at an angle wrt (with respect to) the original direction of motion. The components of momenta in the perpendicular direction cancel each other out, since there was no momentum in the perpendicular direction before the collision. But before we explore momentum transfers more closely, we want to examine collisions in general. We especially want to bring energy conservation as well as momentum conservation into the analysis, so we can use these powerful conservation laws together. We will see that together, these conservation laws enable us to answer many (if not most) questions that arise in collisions, whether they be collisions of cars or galaxies or the elementary particles physicists study in the collisions in particle accelerators. And we can do this without having to know any details of the actual forces that act during the collision or the details of how the motion actually changed during the collision. That is, we do it with a “before and after” approach, not a detailed analysis of the forces and motion approach, which we will take up later in this chapter. With the combination of energy and momentum conservation, we have an extremely powerful and general method of analyzing many physical phenomena. There are, however, some important questions that can’t be answered without using a detailed analysis of forces and motion.

Collisions: Momentum & Energy Conservation

We just saw that if the external forces are negligible in a collision, the total momentum is conserved. What about energy? If, during a collision, kinetic energy is not converted to thermal energy or into deforming the objects (bond energy), then kinetic energy is also conserved. That is, the kinetic energy of the system before the collision will equal the kinetic energy after the
collision. When two cars crash into each other, kinetic energy is usually not conserved (unless they are “bumper cars” with heavy spring bumpers that convert kinetic energy to spring (elastic) potential energy and then back to kinetic energy). In the example shown in the previous figure, the kinetic energy just before the collision is much greater than the kinetic energy immediately after the collision. We can see this, by recognizing that kinetic energy is proportional to the square of the magnitude of the momentum vector (length of the vector). The magnitudes of the two arrows representing initial momenta are much longer than the magnitudes of the two arrows representing final momenta. Much of the kinetic energy must have gone into deformation of the cars and into thermal energy during the collision. If the collision is between two protons or two billiard balls, kinetic energy might be exactly or almost conserved. Collisions in which kinetic energy is conserved are called elastic collisions.

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Elastic Collisions

In an elastic collision between two objects (particles) both the momentum and kinetic energy are conserved. That is, the values of the total momentum and kinetic energy of the system before the collision are equal to the values they have after the collision. This gives us an equation relating the squares of the speeds of the objects from KE conservation, and a vector equation (one equation for each dimension) relating the velocity components from linear momentum conservation.

\[
KE_i = KE_f \quad and \quad p_{toti} = p_{totf} \quad \text{(label 7.2.1)}
\]

As an example, consider the one-dimensional collision of two identical billiard balls. Suppose the first ball is at rest and is hit “head on” by a second ball which has velocity \(v\). What are the velocities of the two balls after the collision? Both momentum and energy conservation hold so both equations of 7.2.1 must be satisfied. Writing them out in detail we have:

\[
\frac{1}{2} m v_{2i}^2 = \frac{1}{2} m v_{2f}^2 + \frac{1}{2} m v_{1f}^2
\]

\[
m v_{2i} = m v_{2f} + m v_{1f}
\]

The only solution of these two simultaneous equations is

\[
v_{1f} = v_{2i}
\]

and

\[
v_{2f} = 0
\]

That is, the second ball comes to rest and the first moves off with the same velocity the second ball had initially. This is illustrated in the accompanying Figure 7.2.1.
If the masses are not equal, both objects will have non-zero velocity after the collision. When the collision involves motion in more than one dimension, we can write a momentum conservation equation for each component of the total momentum. The algebra might get a little messy, but the idea is pretty straightforward.

**Inelastic Collisions**

In an inelastic collision between two objects kinetic energy is not conserved, so we can not equate initial and final kinetic energies. However, an interesting special case occurs when the collision is “completely inelastic” so that the objects stick together. Then they both have the same final velocity after the collision.

Example: Calculating Velocity and Change in Kinetic Energy: Inelastic Collision of a Puck and a Goalie

(a) Find the recoil velocity of a 70.0-kg ice hockey goalie, originally at rest, who catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. (b) How much kinetic energy is lost during the collision? Assume friction between the ice and the puck-goalie system is negligible. (See Figure )

**Figure 7.2.3.** An ice hockey goalie catches a hockey puck and recoils backward. The initial kinetic energy of the puck is almost entirely converted to thermal energy and sound in this inelastic collision.
Strategy

Momentum is conserved because the net external force on the puck-goalie system is zero. We can thus use conservation of momentum to find the final velocity of the puck and goalie system. Note that the initial velocity of the goalie is zero and that the final velocity of the puck and goalie are the same. Once the final velocity is found, the kinetic energies can be calculated before and after the collision and compared as requested.

Solution for (a)

Momentum is conserved because the net external force on the puck-goalie system is zero.

Conservation of momentum is

\[ p_1 + p_2 = p'_1 + p'_2 \]

or

\[ m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2. \]

Because the goalie is initially at rest, we know \( v_2 = 0 \). Because the goalie catches the puck, the final velocities are equal, or \( v'_1 = v'_2 = v' \). Thus, the conservation of momentum equation simplifies to

\[ m_1v_1 = (m_1 + m_2)v'. \]

Solving for \( v' \) yields

\[ v' = \frac{m_1}{m_1 + m_2}v_1. \]

Entering known values in this equation, we get

\[ v' = \frac{0.150 \text{ kg}}{70.0 \text{ kg} + 0.150 \text{ kg}} \times 35.0 \text{ m/s} = 7.48 \times 10^{-2} \text{ m/s}^2 \]

\[ = 0.196 \text{ J}. \]

The change in internal kinetic energy is thus

\[ KE'_\text{int} - KE_\text{int} = 0.196 \text{ J} - 91.9 \text{ J} \]

\[ = -91.7 \text{ J} \]

where the minus sign indicates that the energy was lost.

Discussion for (b)

Nearly all of the initial internal kinetic energy is lost in this perfectly inelastic collision. \( KE_\text{int} \) is mostly converted to thermal energy and sound.

During some collisions, the objects do not stick together and less of the internal kinetic energy is removed—such as happens...
in most automobile accidents. Alternatively, stored energy may be converted into internal kinetic energy during a collision. Figure shows a one-dimensional example in which two carts on an air track collide, releasing potential energy from a compressed spring. Example deals with data from such a collision.

Figure 7.2.4. An air track is nearly frictionless, so that momentum is conserved. Motion is one-dimensional. In this collision, examined in Example, the potential energy of a compressed spring is released during the collision and is converted to internal kinetic energy.

Collisions are particularly important in sports and the sporting and leisure industry utilizes elastic and inelastic collisions. Let us look briefly at tennis. Recall that in a collision, it is momentum and not force that is important. So, a heavier tennis racquet will have the advantage over a lighter one. This conclusion also holds true for other sports—a lightweight bat (such as a softball bat) cannot hit a hardball very far.

The location of the impact of the tennis ball on the racquet is also important, as is the part of the stroke during which the impact occurs. A smooth motion results in the maximizing of the velocity of the ball after impact and reduces sports injuries such as tennis elbow. A tennis player tries to hit the ball on the “sweet spot” on the racquet, where the vibration and impact are minimized and the ball is able to be given more velocity. Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

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