2. Magnetic Forces

Until now we have talked about our everyday experience of magnetic fields originating from what are called permanent magnets. But magnetic fields and electric charges are intimately linked, as we will soon see in greater detail. For now we will switch gears and consider how a charged particle behaves in a magnetic field, and in particular what force it feels.

We do not really care how the magnetic field got there just yet, and for simplicity’s sake we will imagine that the field was created by a magnet. This ignores the interesting question of what makes a magnet (a question we return to later). In other words, we are going to start with the field model of magnetic fields: \[
\text{Irrelevant Thing} \xrightarrow{\text{creates}} \text{Field } \mathbf{B} \xrightarrow{\text{exerts force on}} \text{Charge } q
\]
Magnetic forces are, of course, vectors with both magnitude and direction. We will begin by analyzing their magnitude, and for now put aside the discussion of direction for "Right-Hand Rule".

Magnetic Force on a Moving Charge

Experiments demonstrate that the magnetic force exerted on a charged particle depends on its velocity. If a charge is not moving, it feels no force from the magnetic field! The magnitude of the magnetic field on a charge \(q\) traveling with velocity \(\mathbf{v}\) is given by \[| \mathbf{F}_{\mathbf{B} \text{ on } q} | = |q| |\mathbf{v}| |\mathbf{B}| |\sin \theta|\] where \(\theta\) is the angle between the vectors velocity \(\mathbf{v}\) and magnetic field \(\mathbf{B}\). We've taken the absolute value of every quantity in the above formula because for now we're only considering magnitude.

Notice that if the velocity \(\mathbf{v}\) and the magnetic field \(\mathbf{B}\) point in the same direction or in opposite directions, then \(\theta\) takes a value that gives \(\sin \theta=0\) and the magnetic force is zero. In fact, to calculate the magnetic force all we need to know is the component of velocity perpendicular to the \(\mathbf{B}\) field. This is why we include the \(\sin \theta\) term in the above formula. Another way of rewriting the force from the magnetic field is \[\]|
This is very similar to calculations of the magnitude of torque from 7B, in which only the component of force perpendicular to the lever arm mattered. Below, both approaches for calculating magnetic force on the same setup are shown.

### Right-Hand Rule

When we looked at gravitational fields, we learned that the gravitational force was always in the same direction as the field. When looking at the electric field, we learned that the force for positive and negative charges was with the field and against the field, respectively. The magnetic field is quite different; the force on a charged particle never points in the direction of the magnetic field.

We noted that the velocity \(\mathbf{v}\) of the charged particle determined the magnitude of the magnetic force. It is also needed to determine the direction of the magnetic force. Experimentally, we find that the magnetic force on a test charge is at 90° to the magnetic field \(\mathbf{B}\), and 90° to the velocity of the test charge \(\mathbf{v}\). If the magnetic field \(\mathbf{B}\) and velocity \(\mathbf{v}\) are not pointing in the same direction, then there are only two possible vectors that are at 90° to both of these directions. One of these directions is the magnetic force on a positively-charged particle; the other is the direction of the force on a negatively-charged particle. This illustration should make things more clear (you may want to review the appendix on vector conventions):

To correctly pick the direction for the positive particle, we use right hand rule #2 (RHR #2). Point your right thumb in the direction the particle is going (\(\mathbf{v}\)), your right index finger in the direction that the \(\mathbf{B}\) field is going, and then our middle finger on our right hand when bent will point in the direction of the magnetic force. The mnemonic “very bad finger” and the diagram below may help you remember the order:
Compare the hand figure with the pictures above it, and you'll see that RHR #2 gives us the direction a positively charged particle would feel from the field. To perform RHR for a negative charge, find the direction of the magnetic force on a positive charge and reverse it.

All of the above was done under the assumption that the magnetic field \( \mathbf{B} \) and the velocity \( \mathbf{v} \) were in different directions. How do we determine the direction if we decide to fire a charge in exactly the same direction as the magnetic field? The answer is it does not matter. As we learned before, if the velocity and magnetic field are in exactly the same (or opposite) direction then the magnitude of the magnetic force is zero! As we saw before, if the \( \mathbf{B} \) field and the velocity \( \mathbf{v} \) are parallel (or anti-parallel), the angle between them \( \theta \) is an integer multiple of \( \pi \); therefore \( \sin \theta \) is also zero, and the force is zero.

With both the direction and the magnitude determined, we can now summarize the magnetic force on a test charge \( q \):

\[
\mathbf{F}_{\mathbf{B} \text{ on } q} = \begin{cases} \text{Magnitude} & = |q||\mathbf{v}||\mathbf{B}||\sin \theta| = |q||\mathbf{v}_\perp||\mathbf{B}| \\ \text{Direction} & = \text{ Use RHR if } q \text{ is positive, swap direction if } q \text{ is negative} \end{cases}
\]

Magnetic Force on a Wire Carrying Current

Regardless of whether a charge is in a vacuum or inside a material, it will experience a force when it moves across magnetic field lines. Therefore, a wire with a current (composed of many individual moving charges) can also feel a force when placed in a magnetic field. That force is the vector sum of all the forces acting on the charge carriers that are individually moving in the wire.

Consider a straight wire segment of length \( L \) with a current \( I \) flowing from left to right, placed on the page. Imagine that there is a \( \mathbf{B} \) field in this region that makes an angle \( \theta \) with respect to the wire. If the charges in the wire are moving at an average speed \( \mathbf{v} \), the time they need to travel the length \( L \) is \( t = L/\mathbf{v} \). The amount of charge that flows in this time is \( q = It = IL/\mathbf{v} \). Therefore the force exerted on the wire is
The direction of the magnetic force on a wire is also given by the same right-hand rule used for single charges. However, charges moving in the wire are electrons (negatively-charged), and they travel against the direction of \( I \), which is the movement of positive charge by convention. However, we find that performing RHR on the current or the electrons separately will give us the same answer.