4. Magnetic Induction

What Is Induction?

We have seen how Ørsted was able to demonstrate that electric currents can produce magnetic fields. The English physicist Michael Faraday, a brilliant experimentalist, was the first to demonstrate the converse effect: magnetic fields can be used to induce electric currents. This is now called the principle of magnetic induction. It is interesting to note that Faraday had little formal schooling, so mathematics was by no means his strength. Nevertheless, he was one of the most influential scientists not just of his time, but his contributions continue to find applications to this day.

For example, when he demonstrated that magnetic fields could be used to produce a current in a wire loop, politicians were not impressed as they failed to see the use of it. It turns out that this was the critical step in creating generators and power plants, which make electricity available without flying a kite in a storm or carrying large arrays of batteries. The alternating-current circuits that power all the electrical grids of the world have as part of their components a generator that is based on magnetic induction.

More recently, highly fuel efficient vehicles such as the gas-electric hybrid cars employ a technology called regenerative braking. This uses a device that can give power to the wheels of the car by means of an electric battery, and can recharge the battery during braking by running the circuit in “reverse”, transforming the kinetic energy of the car’s motion into electric potential energy stored in the battery and saving fuel as a result. This recent application has large implications for the world’s economy and is of global environmental impact, and at its core lies Faraday’s principle of induction: that we can transform magnetic fields to electric currents.
Magnetic Flux

Before we tackle the actual form of the principle of magnetic induction, we first need to define a quantity which is crucial to understand it quantitatively: the concept of magnetic flux.

Generalizing Flux

Let us discuss first the idea of flux in general using a familiar example: rain falling on the windshield of a car. Let us suppose that we want to quantitatively determine the amount of rain that hits the windshield of the car. For simplicity, let us first assume that the rain is falling vertically down, and that the shape of the windshield is a rectangle. Let us further simplify by assuming you are in a parked car, i.e. it is not moving. If we want to find how much rain hits the windshield, we need to consider chiefly these three variables:

- The amount of rain
- The size of the windshield
- The orientation of the windshield relative to the rain

Let’s discuss each in turn. If it is raining hard, there will be a lot more raindrops hitting the windshield than if it is raining lightly. Likewise, if the size of the windshield is large, more raindrops will hit it than if it were small. The orientation between the rain and the windshield will also determine how much rain hits the windshield; if the windshield were arranged vertically, there would be no rain hitting the windshield (in the idealization that the windshield is infinitely thin). Conversely, the most amount of rain will hit if it's arranged perpendicular to the rain, or horizontally (like the sunroof on top of the car).

Definition of Magnetic Flux \( \Phi \)

This idea of calculating the amount of rain hitting a surface can be generalized by the concept of flux. The flux of a given quantity through a given surface area is a measure of how much of that quantity passes through the area. We can consider the flux of a vector field passing through an arbitrarily-chosen surface. In our example, the rain falling vertically is our vector field, and the windshield is our arbitrary area. More rain means that the magnitude of the vector field increases. A larger windshield means a greater surface area. The orientation is measured by the angle between the direction of the vector field and the vector normal to the surface area. For the particular case of the magnetic field vector \( \mathbf{B} \), we define the magnetic flux \( \Phi \) through an area \( A \) as \( \Phi = \mathbf{B} \cdot \mathbf{A} = |\mathbf{B}| |A| \cos \theta \) where the angle \( \theta \) is the angle between the magnetic field vector \( \mathbf{B} \) and the vector normal to the surface area \( A \).

From our rain example, you can see that when the rain is falling vertically down, and the windshield surface is horizontal, the vector normal to the area will be vertical. Hence, the angle in the equation will be \( \theta = 0^\circ \), and \( \cos \theta = 1 \) leading to a large flux. If the windshield surface is vertical, the vector normal to the surface area will now be horizontal, the angle will be \( \theta = 90^\circ \), so \( \cos \theta = 0 \) and the flux will vanish (no rain hits the vertical windshield in our example). Note that for an open surface such as a windshield we still have the freedom to choose the normal vector on either side of the surface. This will have no physical effect, but will simply change the value of the angle by 180°. In other words, the flux as we defined
it will change sign. We will see shortly that, physically, changes in flux are more important than the actual value of the flux. So any of the choices we make for convention will lead to identical changes in flux, resolving any ambiguity.

To summarize, the variables of interest when calculating the magnetic flux through an area will be:

- The magnitude of the $\mathbf{B}$ field
- The size of the area under study.
- The angle between the $\mathbf{B}$ field vector and the vector normal to the area.

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**Faraday's Law of Induction**

Now that we have defined the magnetic flux $\Phi$, we can describe Faraday’s observations quantitatively. He sought out to describe a connection between the magnetic field and the current of a wire in the presence of the field. For a current to flow, we must have a closed wire loop (a circuit). The area we will consider for magnetic flux will be the area enclosed by our wire loop. Note that the wire can be looped in a circular, square, or arbitrarily complicated shape.

Considering the magnetic flux through a wire loop, Faraday asked what happened if you placed a magnet close to the loop and let it sit there. Would a current appear in the presence of the magnet? He carried out the experiment and found that there was no current in the loop. However, if you move the magnet away, then for a brief instant a current appears. If you move it back, then a current appears.

What Faraday found is there is an induced current (and therefore induced voltage) only when the magnetic flux changes over time. We say that the current is "induced" because it's not created by a battery, or some connected voltage source like we've seen before. The current is induced in the wire by the magnetic field. He called the induced voltage the induced “electromotive force”, or induced EMF for short, denoted by $\mathcal{E}$ (you can still find it under this name in many textbooks). We therefore refer to his findings as Faraday’s Law of Magnetic Induction. Specifically what he found was that:

- The induced voltage $\mathcal{E}$ is proportional to the rate of change of the flux with time, $\Delta \Phi / \Delta t$.
- If you add loops to the wire coil, each loop will contribute equally to $\mathcal{E}$; if you have $N$ coils, the induced voltage will be $N$ times as strong.

We now summarize these findings in the equation that embodies Faraday's Law: $\mathcal{E} = -N \frac{\Delta \Phi}{\Delta t}$. This means is that you need to have a changing magnetic flux to produce an induced voltage. If the magnetic flux does not change with time, then there will be no current. Only if the magnetic flux changes with time will we observe a current. Furthermore, the faster the flux changes, the larger the induced voltage. You can picture this last statement in the following way. If you are inducing current by moving a magnet close to a wire, the current will be larger if you move the magnet quickly than if you move it slowly. The magnitude of the rate of change is proportional to the voltage; the faster the magnetic field changes, the greater the induced current and induced voltage.

Note also that Faraday’s law focuses only on the effect of a changing magnetic field on a wire. For simplicity, we discussed using a permanent magnet as the source of our field. However, we could also use the magnetic field produced by current in another wire. In fact, this is how Faraday studied induced current and induced voltages.
Lenz's Law: The Direction of Induced Current

The equation presented above for the EMF \(\mathcal{E}\), derived from Faraday’s Law, also has a sign. What is its significance? The sign gives the direction of the induced current in the loop. So far we have not discussed how we are to choose between the two possibilities for the current's direction. Experimentally, if we change a magnetic flux to induce a current, the current will flow to produce a new flux that opposes our change. This is known as Lenz’s Law. We can illustrate how this works with some examples.

Consider a circular loop of wire on a fixed plane. There is no magnetic field in the region of the wire. At \(t = 0\) we create a magnetic field (doesn't matter how) pointing straight out of the plane. We make the magnitude of the field increase linearly with time. From Faraday’s Law, we know that there will be an induced voltage, because the flux through the circuit is increasing with the field. The induced voltage will be zero for \(t < 0\) because the \(\mathbf{B}\) field doesn't change then. For \(t > 0\), the flux is non-zero.

The area enclosed by the circuit is constant. The angle between the normal to the area of the circle and the \(\mathbf{B}\) field is constant \(\theta = 0\), so \(\cos \theta = 1\) and is constant. The magnitude of the \(\mathbf{B}\) field increases with time. Therefore, the changing magnetic field is the only contribution to the change in flux. Recall that if the magnitude increases linearly with time, the rate of change (the slope) of the \(\mathbf{B}\) field magnitude vs. time is a constant (mathematically, \(\frac{\Delta \Phi}{\Delta t} \propto \frac{\Delta \mathbf{B}}{\Delta t} = \) constant). Therefore the induced voltage will be constant.

Will the current flow clockwise or counterclockwise? Now we use Lenz’s Law. To consider flux, we choose the normal going out of the page, in the same direction of the \(\mathbf{B}\) field. This means the flux is positive and it increases with time. The induced current should oppose this; it should produce a negative magnetic flux through the loop that tries to cancel the positive flux. We previously established a positive flux means the field points out of the page. We see that the \(\mathbf{B}\) field produced by the induced current, which we call the induced field \((\mathbf{B}_{\text{ind}})\), should then be pointing into the page. Now use RHR; if you align your thumb along the tangent to our circuit, and curl your fingers towards \(\mathbf{B}_{\text{ind}}\), your thumb will indicate the direction of the induced current. With \(\mathbf{B}_{\text{ind}}\) pointing into the page, we see that the induced current for this case is flowing in a clockwise direction.

Exercise

A similar analysis as above should be carried out for each induced current you encounter. You can try it by yourself and figure out the current direction in the following variations on the above example:

- No field before \(t = 0\), increasing \(\mathbf{B}\) field pointing into the page for \(t > 0\).
- A constant \(\mathbf{B}\) field pointing out of the page before \(t = 0\) and decreasing linearly with time from \(t > 0\).
- Starting with zero magnitude at \(t = 0\), \(\mathbf{B}\) increases linearly with time up to a maximum magnitude at \(t = T/2\). It then decreases linearly with time down to zero magnitude at \(t = T\). This repeats; the period is \(T\) seconds.
Real World Applications of Faraday's Law

We started discussing Faraday’s law by considering moving a magnet near a loop of wire. We have found that this produces an induced current in the wire. This phenomenon has found many familiar applications in the modern world:

- **Seismograph:** One way to exploit Faraday’s Law is to attach a magnet to anything that moves and place it near a loop of wire; any movement or oscillation in the object can be detected as an induced current in the wire loop. In this way we can translate physical movements and oscillations into electrical impulses. In all devices of this kind, the movement or oscillation is measured between the position of a coil relative to a magnet, whose movement causes the current in the coil to vary, generating an electrical signal. For example, as the vibrations produced by an earthquake pass through a seismograph, a magnet's vibrations produce a current that can be amplified to drive a plotting pen. This is how the seismograph operates.

- **Guitar Pickup:** Les Paul, a pioneer musician of pop-jazz guitar, applied Faraday’s Law to the making of musical instruments and invented the first *electric guitar*. The “pickup” of an electric guitar consists of a permanent magnet with a coil of wire wrapped around it several times. The permanent magnet is placed very close to the metal guitar strings. The magnetic field of the permanent magnet causes a part of the metal string of the guitar to become magnetized. When one plucks the string, it vibrates, creating a changing magnetic flux through the coil of wire surrounding the permanent magnet. The coil "picks up" the vibrations that generate an induced current and sends the signal to an amplifier, to the pleasure of rock fans everywhere.

- **Electric Generator:** An electric generator is used to efficiently convert mechanical energy to electrical energy. The mechanical energy can be provided by any number of means, such as falling water (like in a hydroelectric generator), expanding steam (as in coal, oil, and nuclear power plants), or wind (as in wind turbine generators). In all cases, the principle is the same, the mechanical energy is used to move a conducting wire coil inside a magnetic field (usually by rotating the wire). In this case, the area of the coil is the constant, the magnitude of the field is constant, so the angle term \(\cos \theta\) in our equation in Faraday’s law is responsible for the changing. This is caused by the change in the relative orientation between the magnetic field and the normal to the area of the coil. Consider the simple scenario where we rotate the coil with constant angular speed \(\omega\). The rotation angle is given by \(\theta = \omega t\), and the flux will be proportional to \(\cos \omega t\). From differentiable calculus, the time rate of change of the flux will then be proportional to \(\omega \sin \omega t\). This means the induced current will oscillate sinusoidally. In other words, the current in the coil alternates in direction, flowing in one direction for half the cycle and flowing the other direction for the other half. This kind of generator is referred to as an *alternating current generator*, or simply an AC generator. The standard plugs you use to power all of your electrical appliances are all powered by an electric generator of this form.

- **Electric Motor:** Electric motors work in basically the reverse principle that operates electric generators: an alternating electric current causes an electromagnetic cylinder to periodically switch poles, which interacts with the field of an inlaid magnet to turn it. Some motors use electromagnets for both components, but the principle is the same. The stationary magnetic piece is called the *stator* and the magnetic piece that rotates is called that *rotor*.

- **Hybrid Cars:** Regenerative breaking.

Hands-On

In the simulation below, drag the bar magnet around to alter the flux of magnetic field lines through the wire loop (there's an option to help you visualize the field lines). The light attached to the wire loop will glow if the voltage caused by change in flux is high enough. Notice that the bulb stops glowing if you stop moving the magnet, because only the amount of change of
field lines has an effect on voltage. Try to predict how the voltage will respond to different motions you make with the magnet. Once you have a good idea, try to predict how adding a second wire loop will change the light's behavior.

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