1. Harmonic Electromagnetic Waves

When we started discussing the electric and magnetic fields they seemed to be quite separate. We knew that any electric charge (moving or not) created an electric field, and that any moving charge created a magnetic field. Other than that there is not much similarity: the forces produced by the electric field look very different than the forces produced by the magnetic field, and there are no pieces of "magnetic charge" (monopoles) for magnetic field lines to start or end on. Then we started to discuss induction; we discovered that changing the magnetic flux through a loop caused an electric current to flow. By calculating the force on the moving charges within a wire we could show that we got a current only when the magnetic flux through the loop changed.

But one detail has been ignored in the preceding analysis. We know a change in flux, and hence a change in current, can be caused by increasing or decreasing the magnetic field. If we model our charges as starting at rest, then \(v = 0\) and the magnetic field (changing or not) does not seem capable of forcing them to move. Recall that magnetic forces should only exist for moving charges, as \(|q| \mathbf{F}_{\mathbf{B} \text{ on charge}} = |q| \mathbf{B} \times \mathbf{v} \sin \theta = 0\). So How does the current start? The answer to this question is that changing a magnetic field produces an electric field. The electric field so produced does not begin or end on charges; instead it connects with itself so that the field lines don't start or end (similar to the magnetic field). We should emphasize that even though these two methods of creating a current (creating forces on charges in the wire, or creating an electric field with a changing magnetic field) seem very different, for either of them or any combination of the two, the method of calculating the voltage by looking at the change in flux works.

The fact that a changing magnetic field creates an electric field suggests that the electric and magnetic fields are more closely related than we originally thought. In fact, the rules of electromagnetism are inconsistent as we currently know them! If we kept only the rules we knew, the answers to some of our calculations would depend on how we choose to calculate them! The change that we make here (and shown experimentally to be correct) is: a changing electric field creates a magnetic field.
If we accept this new rule, an interesting possibility arises. If we have a magnetic field that is changing, we can create an electric field. If that electric field changes, it can create a magnetic field. We can imagine a situation where we start a magnetic field going, and then it creates an electric field which is itself changing. This field creates a changing magnetic field, which creates a changing electric field, which creates a changing magnetic field, and so on. A proper mathematical treatment shows that not only can these disturbances occur, but that these disturbances do not happen in the same place – rather they propagate, and travel like material waves. We call these propagating disturbances in the electric and magnetic fields electromagnetic waves. They're more colloquially referred to as light.

While there are many different types of electromagnetic waves, including pulse waves and spherical waves, we will devote our attention to the plane wave. The reason is a practical one; at long distances from the source, wavefronts of most electromagnetic waves look flat, and the wave can be approximated as a plane wave. For electromagnetic waves, the \( \mathbf{E} \) and \( \mathbf{B} \) fields oscillate sinusoidally, as harmonic waves. Each has a harmonic wave equation:

\[
\mathbf{E}(x,t) = E_0 \sin \left( \frac{2 \pi t}{T} \pm \frac{2 \pi x}{\lambda} + \phi \right) \hat{e} \\
\mathbf{B}(x,t) = B_0 \sin \left( \frac{2 \pi t}{T} \pm \frac{2 \pi x}{\lambda} + \phi \right) \hat{b}
\]

The difference between these equations and harmonic equations from earlier are \( \hat{e} \) and \( \hat{b} \) at the end. These are vectors that specify the direction that the electric and magnetic fields oscillate. The vector \( \hat{e} \) is a unit vector, so its magnitude is \( |\hat{e}| = 1 \); it points along the direction in which the electric field is oscillating. Similar remarks apply for the magnetic field and its unit vector \( \hat{b} \).

The directions \( \hat{e} \) and \( \hat{b} \) are related; for an electromagnetic wave traveling through free space the electric and magnetic fields oscillate perpendicular to one another, also perpendicular to the direction that the wave travels. If you hold your thumb, index and middle fingers perpendicular to one another you can always point your thumb in the direction of the \( \mathbf{E} \) field, your index finger in the direction of the \( \mathbf{B} \) field, and your middle finger will point in the direction that the wave is traveling. A model of an electromagnetic wave is shown below.

Because the oscillations in both the \( \mathbf{E} \) and \( \mathbf{B} \) fields are perpendicular to the direction of motion, an electromagnetic wave is a transverse wave. The plane of polarisation is the plane containing the direction in which electric
field oscillates and the direction the wave travels (this is the plane containing $\hat{\mathbf{e}}$ and $\hat{\mathbf{k}}$ in the picture above).

The model above also demonstrates some characteristics found in the harmonic equations for the $\mathbf{E}$ and $\mathbf{B}$ fields. Notice that the $\mathbf{E}$ and $\mathbf{B}$ fields have the same wavelength $\lambda$. They are also in phase, and so they must have the same phase constant $\phi$. That is why, unlike before, the $\phi$ and $\lambda$ don’t have subscripts to specify which wave they reference. Furthermore, the disturbances in the $\mathbf{E}$ and $\mathbf{B}$ fields travel with the same speed, which tells us that the period $T$ must be the same. Lastly, the amplitudes of the electric and magnetic fields, $E_0$ and $B_0$, are related. For stronger magnetic fields, oscillations in the field cause a larger change in field, so we expect stronger electric fields too. This is in fact the case. The relationship between the amplitudes of the electric and magnetic field oscillations is $E_0 = c B_0$ where $c$ is the speed of light (approximately $3 \times 10^8 \text{ m/s}$).

Exercise

If an electromagnetic wave had its electric field pointing to the right of a page, and the magnetic field pointing to the top of the page, which way would the electromagnetic wave be traveling?