8.1: The Newtonian Model

The modern understanding of forces and the detailed relation of forces to motion was worked out by Newton and summarized in what has become known as “Newton's Laws of Motion.” We have already spent considerable time making sense of Newton’s first and third laws, which are really part of our understanding of forces. The heart of this chapter, Newton’s second law, tells us how motion changes in time as a result of an unbalanced force. Be sure to refer back to Chapter 6 as frequently as necessary to refresh your understanding of forces, net force, and Newton’s 1st and 2nd laws.

Newton’s 2nd Law

When the forces (torques) don't balance, the relation between the unbalanced force \(\sum F\), and the degree of change of motion is given by Newton's Second Law. As you might surmise, when \(\sum F \neq 0\), there will be a change in the motion. From Chapter 7, we know that an unbalanced force acting over a time produces a change in the momentum of the object:

\[
\int \sum F \, dt = \Delta p
\]

Newton’s 2nd Law expresses this relationship in terms of the instantaneous time rate of change of momentum:

\[
\sum F = \Delta \left\{ \frac{dp}{dt} \right\}
\]

Since momentum, \(p\), is equal to the product of mass and velocity: \(p = m v\), we can rewrite the previous relation as:

\[
\sum F = \frac{d(mv)}{dt} = m \frac{dv}{dt} = ma
\]
where acceleration, \(a\), is the derivative with respect to time of the velocity, \(v\). For rotation, Newton's 2nd law takes the form:

\[\sum \tau = \frac{dL}{dt} \quad \text{or} \quad \sum \tau = I \alpha\]

Several of the most important points concerning Newton's 2nd law are summarized below:

- An unbalanced force or torque \((\sum F \neq 0 \quad \text{or} \quad \sum \tau \neq 0)\) causes a change in motion of an object.
- The change in motion is actually the time rate of change of velocity or angular velocity:
- The degree of "change of motion," the acceleration, is proportional to the unbalanced force and inversely proportional to the mass of the object.

Some General Comments on Newton's 2nd Law

What is Newton's 2nd Law useful for? What can it tell us? Well, there are two basic situations:

We know the forces that act on an object and we want to know what its motion is.
We know the motion of an object and want to know the details of the forces acting on the object.

The approaches we will develop apply to the motion of all objects in the “classical” realm. That is, they apply to planets orbiting around stars as well as baseballs tossed into the air. This is the physics that NASA uses to know how to fire off a Mars probe that travels millions of miles through space and actually lands on the red (actually a yellowish brown) planet. However, Newton’s 2\textsuperscript{nd} Law is not the physics that describes the motion of the electrons whirring about the nucleus of an atom, or the motion of the neutrons and protons in the nucleus of that atom. Quantum mechanics takes over from Newtonian dynamics when sizes begin to approach atomic dimensions. Newtonian mechanics also gets modified by special relativity when relative speeds of objects become significant compared to the speed of light ($3 \times 10^8 \text{ m/s}$).

Another thing that makes the Newtonian approach rather simple and straightforward (at least compared to Quantum Mechanics), and what makes Newton’s laws so easy to write down, is the simplification that comes from being able to combine the separate microscopic electric and gravitational forces that act between individual atoms into a few macroscopic forces that we model as acting on the entire object at a single point. For example, the gravitational forces that act on each individual atom combine into one gravitational force that acts on the entire object at its center of gravity. The perpendicular contact force is the net result of all of the forces acting between the atoms of the two surfaces that actually are close to each other. We thus reduce the trillion or so individual electric forces that act between the closely spaced atoms on the surfaces to one net contact force.

A warning: It is easy to write down Newton’s 2\textsuperscript{nd} law. Often we can figure out the forces that are acting, and thus get $\sum F$. This immediately gives us the acceleration, $a$. In principle, as we shall see, it is straightforward to get velocity and displacement (and the time required for certain motions to occur) from the acceleration. But in practice, unless we know something about solving differential equations with computers, there are not many examples of motion for which we can easily write down the solution to the differential equation we call Newton’s 2\textsuperscript{nd} law. There are in fact, only three fairly straightforward cases.

1. When the forces are constant, which leads to a constant acceleration.
2. When the forces combine to produce an acceleration that always points toward the same point in space and always has the same magnitude (circular motion), and
3. When the net force is always like the spring force—directly proportional to the displacement of the particle, but in the opposite direction.

This last case leads to the very common oscillatory spring-mass motion we are familiar with from Chapter 3.

In all three of these special cases, it is straightforward to write down simple algebraic expressions for the position of the particle as a function of time. That is, you tell me the time you want to know the position of the particle, and I can use my algebraic equation(s) to predict the position (and velocity and acceleration as well). We will examine the first two cases—constant acceleration and circular motion—in some detail in this chapter and the details of oscillatory motion in Chapter 8. The danger: it is easy to think that these motions are all there is; that Newton’s laws don’t apply to the infinity of other types of motion. They do! It is just that we can’t get nice algebraic expressions for the position of the object as a function of time. But it can always be done with a computer. And we can understand qualitatively what will happen, even if we don’t have a simple algebraic expression to “plug into.”
Finding the Change in Motion from the 2nd Law in a Step-Wise Fashion

Before we look at the two special cases, let’s examine in a little more detail how we find the change in motion using Newton’s second law in a step-by-step fashion. We will make use of the general relationship we have used many times in this text: the new value of some variable is equal to the old value plus the change in that variable. We apply this to all three motion variables, \(\langle r \rangle\), \(\langle v \rangle\), and \(\langle a \rangle\).

\[
[r_f = r_i + \Delta r]
\]

\[
[v_f = v_i + \Delta v]
\]

\[
[a_f = a_i + \Delta a]
\]

The figure 8.1.1 shows these relationships in two-dimensions:

We can interpret these figures this way: If I know the position at the initial time, I can get the position at the final time by adding to the initial position the change in position that occurred during that time interval. Similarly, I can get the velocity at the final time by adding the change in velocity that occurred during the time interval to the initial velocity. Likewise, we get the final acceleration by adding the change in acceleration to the initial acceleration.

How do we find these changes. Well, two of them come directly from the defining relations: The defining relation for the vector \(\langle v \rangle\) is

\[
[v = \frac{dr}{dt}]
\]

We rewrite this to emphasize the change in the position vector (displacement)

\[
[dr = v \Delta t ~~ or ~~ \Delta r = v \Delta t] \]

The second form is exact if the velocity is constant over the time interval \(\langle \Delta t \rangle\), or if we consider \(\langle v \rangle\) to be the average value of the velocity, \(\langle v_{avg} \rangle\), over the time interval, \(\langle \Delta t \rangle\).

Similarly for velocity:

\[
[dv = a \Delta t ~~ or ~~ \Delta v = a \Delta t] \]
Now we invoke Newton’s 2nd law to relate the acceleration \( a \), to the net force:

\[
\sum F = ma \quad \text{or} \quad a = \frac{\sum F}{m}
\]

Now we have a way to step out the motion of an object (modeled as a point particle), if we know the net force that acts on the object. This is illustrated for one time interval, \( \Delta t \) in the figure 8.1.2.

![Figure 8.1.2: Going from net force to a to \( \Delta v \) to \( \Delta r \)](image)

Consistent lengths have been chosen for \( r \), \( v \), and \( a \), so that with a time interval of unity, all three figures can be plotted on the same graph.

Knowing the net force, we can get the acceleration. Knowing the acceleration, we know how the velocity changes. Knowing the change in velocity, we know how the position of the object changes during the time interval. The basic approach outlined above, with only minor refinements, is exactly how Newton’s law is solved using a computer. The relationships expressed above in vector form can, of course, also be expressed in component form.

\[
\begin{align*}
\sum F_x &= ma_x \\
\sum F_y &= ma_y \\
\frac{da_x}{dt} &= v_x(t) \\
\frac{da_y}{dt} &= v_y(t) \\
\frac{dv_x}{dt} &= a_x(t) \\
\frac{dv_y}{dt} &= a_y(t) \\
x(t) &= \int v_x(t) dt + x_0 \\
y(t) &= \int v_y(t) dt + y_0
\end{align*}
\]

These separate sets of \( x \)- and \( y \) equations are completely independent of one another. This is a result of the independence of the spatial dimensions in the Galilean Space-Time Model. The usefulness of this approach—the separation into separate equations for each of the perpendicular directions—is due to the fact that they truly are independent. We can separately treat motion in two dimensions as two one-dimensional problems. For example, a thrown ball, if air friction is negligible, experiences a constant acceleration in the vertical direction due to the gravity force of the Earth pulling down, and zero acceleration in the horizontal direction. Each of these separate motions is straightforward to deal with separately, one dimension at a time..

Whether we work with the vector representations or the component representations depends on the particular questions we are
trying to answer. Sometimes one is more useful; sometimes the other. The component equations are especially useful when there is an obvious difference in the forces (and resulting acceleration) in two perpendicular directions. The vector representation is often useful when the directions of the forces are continually changing and when we want to visualize the total force and total acceleration.

Contributors

- Authors of Phys7B (UC Davis Physics Department)