3. Energy and Intensity of Light

Energy Density of $B$ and $E$ fields

While we have not emphasized it so far, electric and magnetic fields both contain energy. The total amount of energy depends on the values of the fields everywhere, so it is more convenient to define the energy density of the fields. This is the amount of energy per unit volume contained within the fields. Unlike total energy, energy density can be defined easily for specific locations. To obtain it for one position, we only need to know the value of the electric and magnetic fields at that position. This is similar to our motivation for introducing energy densities when we discussed fluids in Physics 7B. The energy density of the electric and magnetic fields are

$$u_E = \frac{1}{2 \mu_0} \frac{E^2}{c^2}$$

$$u_B = \frac{1}{2 \mu_0} B^2$$

where $\mu_0$ is the magnetic permeability of free space, $4 \pi \times 10^{-7}$ N/A$^2$. These results are a little bit tricky to derive from what we already know, so we do not attempt a derivation here. We will just accept this energy density as given.

The definitions for energy density apply both to electromagnetic waves and to static electric and magnetic fields. For electromagnetic waves, both the electric field and the magnetic field contribute to the energy density. The total energy density is the sum of these contributions.

$$u_{\text{total}} = u_E + u_B$$
We know that when the electric and magnetic fields hit zero then the energy density in the fields also goes to zero. The energy density is greatest when the magnetic and electric fields experience their peaks:

\[
\text{u}_{\text{max, EM}} = \text{u}_{\text{E max}} + \text{u}_{\text{B max}}
\]

\[
\text{u}_{\text{max, EM}} = \frac{1}{2 \mu_0} \left( \frac{E_0^2}{c^2} + \frac{B_0^2}{2 \mu_0} \right)
\]

where \(B_0\) and \(E_0\) are the maximum displacement of the magnetic and electric fields, respectively. Recall from the previous section that these two quantities are related by the equation \(E_0 = cB_0\). Rewriting the equations, we find that the maximum energy density of an electromagnetic wave is

\[
\text{u}_{\text{max EM}} = \frac{1}{\mu_0} B_0^2
\]

Energy density at a point along an electromagnetic wave oscillates between 0 and \(\text{u}_{\text{max EM}}\). It's been shown (both mathematically and experimentally) that the average energy density is half of the maximum value \(\text{u}_{\text{EM avg}}\)

\[
\text{u}_{\text{EM avg}} = \frac{1}{2 \mu_0} B_0^2
\]

**Brightness and Intensity**

Typically, light-sensitive devices (like our eyes or film in a camera) are sensitive to the power of light, or how much light energy arrives per unit time. However, the total power given off by a light source is usually not a good indication of how bright something is; if the light source is moved far away, then most of the light will not reach our target, and the light source appears dimmer. Instead we define intensity \(I\) as the power of light going through a unit area (in other words, intensity is the amount of energy that arrives per unit area, per unit time). The more intense the light reaching us is, the “brighter” the light appears to be. To calculate the intensity, consider the box below with a cross-sectional area \(A\):

The amount of light that passes through \(A\) in one second is contained within the box (the length of the box is \(c \times 1 \text{ sec}\)). The average energy density is \(\frac{1}{2} \text{u}_{\text{max}}\). So the total energy passing through the end of the box per second is \(\frac{1}{2} \text{u}_{\text{max}} c A\). Dividing by the area \(A\) we get the intensity \(I\):

\[
I = \frac{1}{2 \mu_0} \left( \frac{E_0^2}{c^2} \right) = \frac{1}{2 \mu_0 c} E_0^2
\]