18.3: Coulomb's Law

Through the work of scientists in the late 18th century, the main features of the electrostatic force—the existence of two types of charge, the observation that like charges repel, unlike charges attract, and the decrease of force with distance—were eventually refined, and expressed as a mathematical formula. The mathematical formula for the electrostatic force is called Coulomb’s law after the French physicist Charles Coulomb (1736–1806), who performed experiments and first proposed a formula to calculate it.

Figure \(\PageIndex{1}\): This NASA image of Arp 87 shows the result of a strong gravitational attraction between two galaxies. In contrast, at the subatomic level, the electrostatic attraction between two objects, such as an electron and a proton, is far greater than their mutual attraction due to gravity. (credit: NASA/HST)

Definition: Coulomb’s Law

Coulomb’s law calculates the magnitude of the force \(F\) between two point charges, \(q_1\) and \(q_2\), separated by a distance \(r\).

\[
F = k \frac{|q_1 q_2|}{r^2}
\]
In SI units, the constant \(k\) is equal to

\[
\text{\(k=8.988 \times 10^9 \ \text{N} \cdot \text{m}^2/\text{C}^2\)} \approx 8.99 \times 10^9 \ \text{N} \cdot \text{m}^2/\text{C}^2.
\]

The electrostatic force is a vector quantity and is expressed in units of newtons. The force is understood to be along the line joining the two charges. (Figure \(\PageIndex{2}\))

Although the formula for Coulomb’s law is simple, it was no mean task to prove it. The experiments Coulomb did, with the primitive equipment then available, were difficult. Modern experiments have verified Coulomb’s law to great precision. For example, it has been shown that the force is inversely proportional to distance between two objects squared \((F \propto 1/r^2)\) to an accuracy of 1 part in \(10^{16}\). No exceptions have ever been found, even at the small distances within the atom.

Example \(\PageIndex{1}\): How Strong is the Coulomb Force Relative to the Gravitational Force?

Compare the electrostatic force between an electron and proton separated by \(0.530 \times 10^{-10} \text{m}\) with the gravitational force between them. This distance is their average separation in a hydrogen atom.

**Strategy**

To compare the two forces, we first compute the electrostatic force using Coulomb’s law, \(F = k \left| \frac{q_1 q_2}{r^2} \right|\). We then calculate the gravitational force using Newton’s universal law of gravitation. Finally, we take a ratio to see how the forces compare in magnitude.

**Solution**

Entering the given and known information about the charges and separation of the electron and proton into the expression of Coulomb’s law yields

\[
\left[ \begin{align*}
F &= k \left| \frac{q_1 q_2}{r^2} \right| \\
&= (8.99 \times 10^9 \ \text{N} \cdot \text{m}^2/\text{C}^2) \times \frac{1.60 \times 10^{-19} \ \text{C} \times 1.60 \times 10^{-19} \ \text{C}}{(0.530 \times 10^{-10} \ \text{m})^2}
\end{align*} \right]
\]

Thus the Coulomb force is

\[
F = 8.19 \times 10^{-8} \ \text{N}.
\]

The charges are opposite in sign, so this is an attractive force. This is a very large force for an electron—it would cause an
acceleration of \(8.99 \times 10^{22} \text{ m/s}^2\) (verification is left as an end-of-section problem). The gravitational force is given by Newton’s law of gravitation as:

\[
F_G = G \frac{mM}{r^2}
\]

where \(G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\). Here \(m\) and \(M\) represent the electron and proton masses, which can be found in the appendices. Entering values for the knowns yields

\[
F_G = (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \times \frac{(9.11 \times 10^{-31} \text{kg})(1.67 \times 10^{-27} \text{kg})}{(0.530 \times 10^{-10} \text{m})^2} = 3.61 \times 10^{-47} \text{N}
\]

This is also an attractive force, although it is traditionally shown as positive since gravitational force is always attractive. The ratio of the magnitude of the electrostatic force to gravitational force in this case is, thus,

\[
\frac{F}{F_G} = 2.27 \times 10^{39}
\]

Discussion

This is a remarkably large ratio! Note that this will be the ratio of electrostatic force to gravitational force for an electron and a proton at any distance (taking the ratio before entering numerical values shows that the distance cancels). This ratio gives some indication of just how much larger the Coulomb force is than the gravitational force between two of the most common particles in nature.

As the example implies, gravitational force is completely negligible on a small scale, where the interactions of individual charged particles are important. On a large scale, such as between the Earth and a person, the reverse is true. Most objects are nearly electrically neutral, and so attractive and repulsive Coulomb forces nearly cancel. Gravitational force on a large scale dominates interactions between large objects because it is always attractive, while Coulomb forces tend to cancel.

Summary

- Frenchman Charles Coulomb was the first to publish the mathematical equation that describes the electrostatic force between two objects.
- Coulomb’s law gives the magnitude of the force between point charges. It is \(F = k \frac{|q_1 q_2|}{r^2}\) where \(q_1\) and \(q_2\) are two point charges separated by a distance \(r\), and \(k \approx 8.99 \times 10^9 \text{N} \cdot \text{m}^2/\text{C}^2\)
- This Coulomb force is extremely basic, since most charges are due to point-like particles. It is responsible for all electrostatic effects and underlies most macroscopic forces.
- The Coulomb force is extraordinarily strong compared with the gravitational force, another basic force—but unlike gravitational force it can cancel, since it can be either attractive or repulsive.
- The electrostatic force between two subatomic particles is far greater than the gravitational force between the same two particles.
Glossary

**Coulomb’s law**
the mathematical equation calculating the electrostatic force vector between two charged particles

**Coulomb force**
another term for the electrostatic force

**electrostatic force**
the amount and direction of attraction or repulsion between two charged bodies

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