22.10: Magnetic Force between Two Parallel Conductors

You might expect that there are significant forces between current-carrying wires, since ordinary currents produce significant magnetic fields and these fields exert significant forces on ordinary currents. But you might not expect that the force between wires is used to define the ampere. It might also surprise you to learn that this force has something to do with why large circuit breakers burn up when they attempt to interrupt large currents.

The force between two long straight and parallel conductors separated by a distance \(r\) can be found by applying what we have developed in preceding sections. Figure \(\PageIndex{1}\) shows the wires, their currents, the fields they create, and the subsequent forces they exert on one another. Let us consider the field produced by wire 1 and the force it exerts on wire 2 (call the force \(F_{2}\)). The field due to \(I_{1}\) at a distance \(r\) is given to be

\[
B_{1} = \frac{\mu_{0}I_{1}}{2\pi r} \label{22.11.1}
\]

\[\text{Figure } \PageIndex{1}: (a) The magnetic field produced by a long straight conductor is perpendicular to a parallel conductor, as indicated by RHR-2. (b) A view from above of the two wires shown in (a), with one magnetic field line shown for the field due to } I_{1} \text{ at a distance } r \text{.} \]

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Each wire. \( \text{RHR-1 shows that the force between the parallel conductors is attractive when the currents are in the same direction. A similar analysis shows that the force is repulsive between currents in opposite directions.} \)

This field is uniform along wire 2 and perpendicular to it, and so the force \( \langle F_2 \rangle \) it exerts on wire 2 is given by \( \langle F = IIB \sin \theta \rangle \) with \( \langle \sin \theta = 1 \rangle \): \( \langle F_2 \rangle = I_2B_i \). By Newton’s third law, the forces on the wires are equal in magnitude, and so we just write \( \langle F \rangle \) for the magnitude of \( \langle F_2 \rangle \). (Note that \( \langle F_1 \rangle = -F_2 \).) Since the wires are very long, it is convenient to think in terms of \( \langle F/l \rangle \), the force per unit length. Substituting the expression for \( \langle B_i \rangle \) into the last equation and rearranging terms gives

\[
\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}.
\]

\( \langle F/l \rangle \) is the force per unit length between two parallel currents \( \langle I_1 \rangle \) and \( \langle I_2 \rangle \) separated by a distance \( \langle r \rangle \). The force is attractive if the currents are in the same direction and repulsive if they are in opposite directions.

This force is responsible for the \textit{pinch effect} in electric arcs and plasmas. The force exists whether the currents are in wires or not. In an electric arc, where currents are moving parallel to one another, there is an attraction that squeezes currents into a smaller tube. In large circuit breakers, like those used in neighborhood power distribution systems, the pinch effect can concentrate an arc between plates of a switch trying to break a large current, burn holes, and even ignite the equipment. Another example of the pinch effect is found in the solar plasma, where jets of ionized material, such as solar flares, are shaped by magnetic forces.

The \textit{operational definition of the ampere} is based on the force between current-carrying wires. Note that for parallel wires separated by 1 meter with each carrying 1 ampere, the force per meter is

\[
\frac{F}{l} = \frac{\left(4\pi \times 10^{-7} \text{T} \cdot \text{m/A} \right) \left( 1 \text{ A} \right)^2}{\left(2\pi\right)\left(1 \text{ m}\right)} = 2 \times 10^{-7} \text{ N/m}.
\]

Since \( \langle \mu_0 \rangle \) is exactly \( 4\pi \times 10^{-7} \text{T} \cdot \text{m/A} \) by definition, and because \( (1 \text{ T} = 1 \text{ N/} \text{A} \cdot \text{m}) \), the force per meter is exactly \( (2 \times 10^{-7} \text{ N/m}) \). This is the basis of the operational definition of the ampere.

Definition: \textit{THE AMPERE}

One ampere of current through each of two parallel conductors of infinite length, separated by one meter in empty space free of other magnetic fields, causes a force of exactly \( (2 \times 10^{-7} \text{ N/m}) \) on each conductor.

Infinite-length straight wires are impractical and so, in practice, a current balance is constructed with coils of wire separated by a few centimeters. Force is measured to determine current. This also provides us with a method for measuring the coulomb. We measure the charge that flows for a current of one ampere in one second. That is, \( (1 \text{ C} = 1 \text{ A} \cdot \text{s}) \). For both the ampere and the coulomb, the method of measuring force between conductors is the most accurate in practice.

\[\text{Summary}\]

- The force between two parallel currents \( \langle I_1 \rangle \) and \( \langle I_2 \rangle \) separated by a distance \( \langle r \rangle \), has a magnitude per unit
length given by \[ \frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}. \]

- The force is attractive if the currents are in the same direction, repulsive if they are in opposite directions.

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