6.4: Friction (Part 1)

Learning Objectives

- Describe the general characteristics of friction
- List the various types of friction
- Calculate the magnitude of static and kinetic friction, and use these in problems involving Newton’s laws of motion

When a body is in motion, it has resistance because the body interacts with its surroundings. This resistance is a force of friction. Friction opposes relative motion between systems in contact but also allows us to move, a concept that becomes obvious if you try to walk on ice. Friction is a common yet complex force, and its behavior still not completely understood. Still, it is possible to understand the circumstances in which it behaves.

**Static and Kinetic Friction**

The basic definition of friction is relatively simple to state.

Friction

Friction is a force that opposes relative motion between systems in contact.

There are several forms of friction. One of the simpler characteristics of sliding friction is that it is parallel to the contact surfaces between systems and is always in a direction that opposes motion or attempted motion of the systems relative to each other. If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction. For example, friction slows a hockey puck sliding on ice. When objects are stationary, static friction can act between them; the static friction is usually greater than the kinetic friction between two objects.
Static and Kinetic Friction

If two systems are in contact and stationary relative to one another, then the friction between them is called static friction. If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you might push very hard on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. If you finally push hard enough, the crate seems to slip suddenly and starts to move. Now static friction gives way to kinetic friction. Once in motion, it is easier to keep it in motion than it was to get it started, indicating that the kinetic frictional force is less than the static frictional force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it easier to get the crate started and keep it going (as you might expect).

Figure \(\PageIndex{1}\) is a crude pictorial representation of how friction occurs at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. Thus, when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, breaking off the points, or both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them. Part of the friction is due to adhesive forces between the surface molecules of the two objects, which explains the dependence of friction on the nature of the substances. For example, rubber-soled shoes slip less than those with leather soles. Adhesion varies with substances in contact and is a complicated aspect of surface physics. Once an object is moving, there are fewer points of contact (fewer molecules adhering), so less force is required to keep the object moving. At small but nonzero speeds, friction is nearly independent of speed.

![Frictional forces](image_url)

Figure \(\PageIndex{1}\): Frictional forces, such as \(\text{vec}(f)\), always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. For the object to move, it must rise to where the peaks of the top surface can skip along the bottom surface. Thus, a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. (In fact, perfectly smooth, clean surfaces of similar materials would adhere, forming a bond called a “cold weld.”)

The magnitude of the frictional force has two forms: one for static situations (static friction), the other for situations involving motion (kinetic friction). What follows is an approximate empirical (experimentally determined) model only. These equations for static and kinetic friction are not vector equations.

Magnitude of Static Friction
The magnitude of static friction \( f_s \) is

\[ f_s \leq \mu_s N, \quad \text{\( \text{\label{6.1}} \)} \]

where \( \mu_s \) is the coefficient of static friction and \( N \) is the magnitude of the normal force.

The symbol \( \leq \) means less than or equal to, implying that static friction can have a maximum value of \( \mu_s N \). Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds \( f_s \) (max), the object moves. Thus,

\[ f_s \text{ (max)} = \mu_s N \]

Magnitude of Kinetic Friction

The magnitude of kinetic friction \( f_k \) is given by

\[ f_k \leq \mu_k N, \quad \text{\( \text{\label{6.2}} \)} \]

where \( \mu_k \) is the coefficient of kinetic friction.

A system in which \( f_k = \mu_k N \) is described as a system in which friction behaves simply. The transition from static friction to kinetic friction is illustrated in Figure \( \PageIndex{2} \).

![Figure \( \PageIndex{2} \): (a) The force of friction \( \vec{f} \) between the block and the rough surface opposes the direction of the applied force \( \vec{F} \). The magnitude of the static friction balances that of the applied force. This is shown in the left side of the graph in (c). (b) At some point, the magnitude of the applied force is greater than the force of kinetic friction, and the block moves to the right. This is shown in the right side of the graph. (c) The graph of the frictional force versus the applied force; note that \( f_s \) (max) > \( f_k \). This means that \( \mu_s \) > \( \mu_k \)](image)

As you can see in Table 6.1, the coefficients of kinetic friction are less than their static counterparts. The approximate values of \( \mu \) are stated to only one or two digits to indicate the approximate description of friction given by the preceding two equations.
Table 6.1 - Approximate Coefficients of Static and Kinetic Friction

<table>
<thead>
<tr>
<th>System</th>
<th>Static Friction $\mu_s$</th>
<th>Kinetic Friction $\mu_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rubber on dry concrete</td>
<td>1.0</td>
<td>0.7</td>
</tr>
<tr>
<td>Rubber on wet concrete</td>
<td>0.5-0.7</td>
<td>0.3-0.5</td>
</tr>
<tr>
<td>Wood on wood</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>Waxed wood on wet snow</td>
<td>0.14</td>
<td>0.1</td>
</tr>
<tr>
<td>Metal on wood</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>Steel on steel (dry)</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>Steel on steel (oiled)</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>Teflon on steel</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Bone lubricated by synovial fluid</td>
<td>0.016</td>
<td>0.015</td>
</tr>
<tr>
<td>Shoes on wood</td>
<td>0.9</td>
<td>0.7</td>
</tr>
<tr>
<td>Shoes on ice</td>
<td>0.1</td>
<td>0.05</td>
</tr>
<tr>
<td>Ice on ice</td>
<td>0.1</td>
<td>0.03</td>
</tr>
<tr>
<td>Steel on ice</td>
<td>0.4</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Equation \ref{6.1} and Equation \ref{6.2} include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force is equal to its weight,

$$w = mg = (100\; \text{kg})(9.80\; \text{m/s}^2) = 980\; \text{N}$$

perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than

$$f_{s} (\text{max}) = \mu_s N = (0.45)(980\; \text{N}) = 440\; \text{N}$$

to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only

$$f_k = \mu_k N = (0.30)(980\; \text{N}) = 290\; \text{N}$$
keeps it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unitless quantity with a magnitude usually between 0 and 1.0. The actual value depends on the two surfaces that are in contact.

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia) and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth, almost-glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint (Figure \(\PageIndex{3}\)). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.

![Figure \(\PageIndex{3}\): Artificial knee replacement is a procedure that has been performed for more than 20 years. These post-operative X-rays show a right knee joint replacement. (credit: Mike Baird)](image)

Natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves. Hospitals and doctor’s clinics commonly use artificial lubricants, such as gels, to reduce friction.

The equations given for static and kinetic friction are empirical laws that describe the behavior of the forces of friction. While these formulas are very useful for practical purposes, they do not have the status of mathematical statements that represent general principles (e.g., Newton’s second law). In fact, there are cases for which these equations are not even good approximations. For instance, neither formula is accurate for lubricated surfaces or for two surfaces siding across each other at high speeds. Unless specified, we will not be concerned with these exceptions.

Example 6.10: Static and Kinetic Friction

A 20.0-kg crate is at rest on a floor as shown in Figure \(\PageIndex{4}\). The coefficient of static friction between the crate and floor is 0.700 and the coefficient of kinetic friction is 0.600. A horizontal force \(\vec{P}\) is applied to the crate. Find the force of friction if (a) \(\vec{P}\) = 20.0 N, (b) \(\vec{P}\) = 30.0 N, (c) \(\vec{P}\) = 120.0 N, and (d) \(\vec{P}\) = 180.0 N.
Figure \(\PageIndex{4}\): (a) A crate on a horizontal surface is pushed with a force \(\vec{P}\). (b) The forces on the crate. Here, \(\vec{f}\) may represent either the static or the kinetic frictional force.

**Strategy**

The free-body diagram of the crate is shown in Figure \(\PageIndex{4b}\). We apply Newton’s second law in the horizontal and vertical directions, including the friction force in opposition to the direction of motion of the box.

**Solution**

Newton’s second law gives

\[
\sum F_x = ma_x \\
\sum F_y = ma_y \\
P - f = ma_x \\
N - w = 0 \dot{\text{d}t}
\]

Here we are using the symbol \(f\) to represent the frictional force since we have not yet determined whether the crate is subject to station friction or kinetic friction. We do this whenever we are unsure what type of friction is acting. Now the weight of the crate is

\[w = (20.0 \; \text{kg})(9.80 \; \text{m/s}^2) = 196 \; \text{N},\]

which is also equal to \(N\). The maximum force of static friction is therefore \((0.700)(196 \; \text{N}) = 137 \; \text{N}\). As long as \(\vec{P}\) is less than 137 N, the force of static friction keeps the crate stationary and \(f_s = \vec{P}\). Thus, (a) \(f_s = 20.0 \; \text{N}\), (b) \(f_s = 30.0 \; \text{N}\), and (c) \(f_s = 120.0 \; \text{N}\). (d) If \(\vec{P} = 180.0 \; \text{N}\), the applied force is greater than the maximum force of static friction (137 N), so the crate can no longer remain at rest. Once the crate is in motion, kinetic friction acts. Then

\[f_k = \mu_k N = (0.600)(196 \; \text{N}) = 118 \; \text{N},\]

and the acceleration is

\[a_x = \frac{\vec{P} - f_k}{m} = \frac{180.0 \; \text{N} - 118 \; \text{N}}{20.0 \; \text{kg}} = 3.10 \; \text{m/s}^2 \dot{\text{d}t}\]

**Significance**

This example illustrates how we consider friction in a dynamics problem. Notice that static friction has a value that matches the applied force, until we reach the maximum value of static friction. Also, no motion can occur until the applied force equals the force of static friction, but the force of kinetic friction will then become smaller.

**Exercise 6.7**

A block of mass 1.0 kg rests on a horizontal surface. The frictional coefficients for the block and surface are \(\mu_s = 0.50\) and \(\mu_k = 0.40\). (a) What is the minimum horizontal force required to move the block? (b) What is the block’s
acceleration when this force is applied?

Contributors and Attributions

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