10.5: Moment of Inertia and Rotational Kinetic Energy

Figure \(\PageIndex{6}\): A boomerang is hurled into the air at an initial angle of 40°.

**Strategy**

We use the definitions of rotational and linear kinetic energy to find the total energy of the system. The problem states to neglect air resistance, so we don’t have to worry about energy loss. In part (b), we use conservation of mechanical energy to find the maximum height of the boomerang.

**Solution**

a. Moment of inertia: \(I = \frac{1}{12} mL^2 = \frac{1}{12}(1.0\; \text{kg})(0.7\; \text{m})^2 = 0.041\; \text{kg} \cdot \text{m}^2\).

Angular velocity: \(\omega = (10.0\; \text{rev/s})(2 \pi) = 62.83\; \text{rad/s}\).

The rotational kinetic energy is therefore \(K_R = \frac{1}{2} I \omega^2 = \frac{1}{2} (0.041\; \text{kg} \cdot \text{m}^2)(62.83\; \text{rad/s})^2 = 80.93\; \text{J}\).

The translational...
kinetic energy is $$K_{T} = \frac{1}{2} mv^{2} = \frac{1}{2} (1.0 \; \text{kg})(30.0 \; \text{m/s})^{2} = 450.0 \; \text{J} \ldotp$$ Thus, the total energy in the boomerang is $$K_{\text{Total}} = K_{R} + K_{T} = 80.93 \; \text{J} + 450.0 \; \text{J} = 530.93 \; \text{J} \ldotp$$

b. We use conservation of mechanical energy. Since the boomerang is launched at an angle, we need to write the total energies of the system in terms of its linear kinetic energies using the velocity in the x- and y-directions. The total energy when the boomerang leaves the hand is $$E_{\text{Before}} = \frac{1}{2} mv_{x}^{2} + \frac{1}{2} mv_{y}^{2} + \frac{1}{2} I \omega^{2} \ldotp$$ The total energy at maximum height is $$E_{\text{Final}} = \frac{1}{2} mv_{x}^{2} + \frac{1}{2} I \omega^{2} + mgh \ldotp$$ By conservation of mechanical energy, $$E_{\text{Before}} = E_{\text{Final}}$$ so we have, after canceling like terms, $$\frac{1}{2} mv_{y}^{2} = mgh \ldotp$$ Since $$v_{y} = (30.0 \; \text{m/s})(\sin 40^\circ) = 19.28 \; \text{m/s}$$, we find $$h = \frac{(19.28 \; \text{m/s})^{2}}{2 (9.8 \; \text{m/s}^{2})} = 18.97 \; \text{m} \ldotp$$

**Significance**

In part (b), the solution demonstrates how energy conservation is an alternative method to solve a problem that normally would be solved using kinematics. In the absence of air resistance, the rotational kinetic energy was not a factor in the solution for the maximum height.

Exercise 10.4

A nuclear submarine propeller has a moment of inertia of 800.0 kg • m$^2$. If the submerged propeller has a rotation rate of 4.0 rev/s when the engine is cut, what is the rotation rate of the propeller after 5.0 s when water resistance has taken 50,000 J out of the system?

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