14.6: Archimedes’ Principle and Buoyancy

Learning Objectives

- Define buoyant force
- State Archimedes’ principle
- Describe the relationship between density and Archimedes’ principle

When placed in a fluid, some objects float due to a buoyant force. Where does this buoyant force come from? Why is it that some things float and others do not? Do objects that sink get any support at all from the fluid? Is your body buoyed by the atmosphere, or are only helium balloons affected (Figure \(\PageIndex{1}\))?  

![Figure \(\PageIndex{1}\): (a) Even objects that sink, like this anchor, are partly supported by water when submerged. (b) Submarines have adjustable density (ballast tanks) so that they may float or sink as desired. (c) Helium-filled balloons tug upward on their strings, demonstrating air’s buoyant effect. (credit b: modification of work by Allied Navy; credit c: modification of work by “Crystl”/Flickr)](image)

Answers to all these questions, and many others, are based on the fact that pressure increases with depth in a fluid. This means that the upward force on the bottom of an object in a fluid is greater than the downward force on top of the object. There is an upward force, or **buoyant force**, on any object in any fluid (Figure \(\PageIndex{2}\)). If the buoyant force is greater than the object’s weight, the object rises to the surface and floats. If the buoyant force is less than the object’s weight, the object sinks.
If the buoyant force equals the object’s weight, the object can remain suspended at its present depth. The buoyant force is always present, whether the object floats, sinks, or is suspended in a fluid.

**Buoyant Force**

The buoyant force is the upward force on any object in any fluid.

![Diagram of buoyant force](image)

**Archimedes’ Principle**

Just how large a force is buoyant force? To answer this question, think about what happens when a submerged object is removed from a fluid, as in Figure 3. If the object were not in the fluid, the space the object occupied would be filled by fluid having a weight $w_f$. This weight is supported by the surrounding fluid, so the buoyant force must equal $w_f$, the weight of the fluid displaced by the object.

**Archimedes’ Principle**

The buoyant force on an object equals the weight of the fluid it displaces. In equation form, **Archimedes’ principle** is

$$F_B = w_f$$

where $F_B$ is the buoyant force and $w_f$ is the weight of the fluid displaced by the object.

This principle is named after the Greek mathematician and inventor Archimedes (ca. 287–212 BCE), who stated this principle long before concepts of force were well established.
Figure \(\PageIndex{3}\): (a) An object submerged in a fluid experiences a buoyant force \(F_B\). If \(F_B\) is greater than the weight of the object, the object rises. If \(F_B\) is less than the weight of the object, the object sinks. (b) If the object is removed, it is replaced by fluid having weight \(w_{fl}\). Since this weight is supported by surrounding fluid, the buoyant force must equal the weight of the fluid displaced.

Archimedes’ principle refers to the force of buoyancy that results when a body is submerged in a fluid, whether partially or wholly. The force that provides the pressure of a fluid acts on a body perpendicular to the surface of the body. In other words, the force due to the pressure at the bottom is pointed up, while at the top, the force due to the pressure is pointed down; the forces due to the pressures at the sides are pointing into the body.

Since the bottom of the body is at a greater depth than the top of the body, the pressure at the lower part of the body is higher than the pressure at the upper part, as shown in Figure \(\PageIndex{2}\). Therefore a net upward force acts on the body. This upward force is the force of buoyancy, or simply \text{buoyancy}.

The exclamation “Eureka” (meaning “I found it”) has often been credited to Archimedes as he made the discovery that would lead to Archimedes’ principle. Some say it all started in a bathtub. To read the story, explore \text{Scientific American} to learn more.

**Density and Archimedes’ Principle**

If you drop a lump of clay in water, it will sink. But if you mold the same lump of clay into the shape of a boat, it will float. Because of its shape, the clay boat displaces more water than the lump and experiences a greater buoyant force, even though its mass is the same. The same is true of steel ships.

The average density of an object is what ultimately determines whether it floats. If an object’s average density is less than that of the surrounding fluid, it will float. The reason is that the fluid, having a higher density, contains more mass and hence more weight in the same volume. The buoyant force, which equals the weight of the fluid displaced, is thus greater than the weight of the object. Likewise, an object denser than the fluid will sink.

The extent to which a floating object is submerged depends on how the object’s density compares to the density of the fluid. In Figure \(\PageIndex{4}\), for example, the unloaded ship has a lower density and less of it is submerged compared with the same ship when loaded. We can derive a quantitative expression for the fraction submerged by considering density. The fraction submerged is the ratio of the volume submerged to the volume of the object, or

\[ \text{fraction submerged} = \frac{V_{\text{sub}}}{V_{\text{obj}}} = \frac{V_{fl}}{V_{obj}} \]

The volume submerged equals the volume of fluid displaced, which we call \(V_{fl}\). Now we can obtain the relationship between
the densities by substituting \(\rho = \frac{m}{V}\) into the expression. This gives

\[
\frac{V_{fl}}{V_{obj}} = \frac{\frac{m_{fl}}{\rho_{fl}}}{\frac{m_{obj}}{\rho_{obj}}},
\]

where \(\rho_{obj}\) is the average density of the object and \(\rho_{fl}\) is the density of the fluid. Since the object floats, its mass and that of the displaced fluid are equal, so they cancel from the equation, leaving

\[\text{fraction; submerged} = \frac{\rho_{obj}}{\rho_{fl}}\]

We can use this relationship to measure densities.

![Figure](image)

Figure: An unloaded ship (a) floats higher in the water than a loaded ship (b).

Example 14.4: Calculating Average Density

Suppose a 60.0-kg woman floats in fresh water with 97.0% of her volume submerged when her lungs are full of air. What is her average density?

**Strategy**

We can find the woman’s density by solving the equation

\[\text{fraction; submerged} = \frac{\rho_{obj}}{\rho_{fl}}\]

for the density of the object. This yields

\[\rho_{obj} = \rho_{person} = (\text{fraction; submerged}) \cdot \rho_{fl} \cdot \text{ldotp}\]

We know both the fraction submerged and the density of water, so we can calculate the woman’s density.

**Solution**

Entering the known values into the expression for her density, we obtain

\[\rho_{person} = 0.970 \cdot 10^3 \text{ kg/m}^3 = 970 \text{ kg/m}^3 \cdot \text{ldotp}\]

**Significance**

The woman’s density is less than the fluid density. We expect this because she floats.
Numerous lower-density objects or substances float in higher-density fluids: oil on water, a hot-air balloon in the atmosphere, a bit of cork in wine, an iceberg in salt water, and hot wax in a “lava lamp,” to name a few. A less obvious example is mountain ranges floating on the higher-density crust and mantle beneath them. Even seemingly solid Earth has fluid characteristics.

**Measuring Density**

One of the most common techniques for determining density is shown in Figure `\(\PageIndex{5}\)`.

![Figure \(\PageIndex{5}\): (a) A coin is weighed in air. (b) The apparent weight of the coin is determined while it is completely submerged in a fluid of known density. These two measurements are used to calculate the density of the coin.](image)

An object, here a coin, is weighed in air and then weighed again while submerged in a liquid. The density of the coin, an indication of its authenticity, can be calculated if the fluid density is known. We can use this same technique to determine the density of the fluid if the density of the coin is known.

All of these calculations are based on Archimedes’ principle, which states that the buoyant force on the object equals the weight of the fluid displaced. This, in turn, means that the object appears to weigh less when submerged; we call this measurement the object’s apparent weight. The object suffers an apparent weight loss equal to the weight of the fluid displaced. Alternatively, on balances that measure mass, the object suffers an apparent mass loss equal to the mass of fluid displaced. That is, apparent weight loss equals weight of fluid displaced, or apparent mass loss equals mass of fluid displaced.

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