3.6: Heat Capacities of an Ideal Gas

Learning Objectives

By the end of this section, you will be able to:

- Define heat capacity of an ideal gas for a specific process
- Calculate the specific heat of an ideal gas for either an isobaric or isochoric process
- Explain the difference between the heat capacities of an ideal gas and a real gas
- Estimate the change in specific heat of a gas over temperature ranges

We learned about specific heat and molar heat capacity previously; however, we have not considered a process in which heat is added. We do that in this section. First, we examine a process where the system has a constant volume, then contrast it with a system at constant pressure and show how their specific heats are related.

Let’s start with looking at Figure 1, which shows two vessels A and B, each containing 1 mol of the same type of ideal gas at a temperature T and a volume V. The only difference between the two vessels is that the piston at the top of A is fixed, whereas the one at the top of B is free to move against a constant external pressure p. We now consider what happens when the temperature of the gas in each vessel is slowly increased to \((T + dT)\) with the addition of heat.
Two vessels are identical except that the piston at the top of A is fixed, whereas that atop B is free to move against a constant external pressure $p$.

Since the piston of vessel A is fixed, the volume of the enclosed gas does not change. Consequently, the gas does no work, and we have from the first law

$$\Delta E_{\text{int}} = dQ - dW = dQ.$$ 

We represent the fact that the heat is exchanged at constant volume by writing

$$dQ = C_v n dT,$$ where $C_v$ is the molar heat capacity at constant volume of the gas. In addition, since $\Delta E_{\text{int}} = dQ$ for this particular process,

$$\Delta E_{\text{int}} = C_v n dT. \label{3.9}$$

We obtained this equation assuming the volume of the gas was fixed. However, internal energy is a state function that depends on only the temperature of an ideal gas. Therefore, $\Delta E_{\text{int}} = C_v n dT$ gives the change in internal energy of an ideal gas for any process involving a temperature change $dT$.

When the gas in vessel B is heated, it expands against the movable piston and does work $dW = pdV$. In this case, the heat is added at constant pressure, and we write $dQ = C_p n dT$, where $C_p$ is the molar heat capacity at constant pressure of the gas. Furthermore, since the ideal gas expands against a constant pressure,

$$d(pV) = d(RnT)$$

becomes

$$pdV = Rn dT.$$ 

Finally, inserting the expressions for $dQ$ and $pdV$ into the first law, we obtain

$$\Delta E_{\text{int}} = dQ - pdV = (C_p n - Rn)dT.$$ 

We have found $\Delta E_{\text{int}}$ for both an isochoric and an isobaric process. Because the internal energy of an ideal gas depends only on the temperature, $\Delta E_{\text{int}}$ must be the same for both processes. Thus,

$$C_v n dT = (C_p n - Rn)dT.$$
and

\[ C_p = C_V + R. \] \label{eq50} \]

The derivation of Equation \ref{eq50} was based only on the ideal gas law. Consequently, this relationship is approximately valid for all dilute gases, whether monatomic like He, diatomic like \((O_2)\), or polyatomic like \((CO_2)\) or \((NH_3)\).

In the preceding chapter, we found the molar heat capacity of an ideal gas under constant volume to be

\[ C_V = \frac{d}{2} R, \]

where \(d\) is the number of degrees of freedom of a molecule in the system. Table \(\PageIndex{1}\) shows the molar heat capacities of some dilute ideal gases at room temperature. The heat capacities of real gases are somewhat higher than those predicted by the expressions of \((C_V)\) and \((C_p)\) given in Equation \ref{eq50}. This indicates that vibrational motion in polyatomic molecules is significant, even at room temperature. Nevertheless, the difference in the molar heat capacities, \((C_p - C_V)\), is very close to \(R\), even for the polyatomic gases.

<table>
<thead>
<tr>
<th>Type of Molecule</th>
<th>Gas</th>
<th>(\langle C_p \rangle) (J/mol K)</th>
<th>(\langle C_V \rangle) (J/mol K)</th>
<th>(\langle C_p - C_V \rangle) (J/mol K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monatomic</td>
<td>Ideal</td>
<td>(\frac{5}{2}R = 20.79)</td>
<td>(\frac{3}{2}R = 12.47)</td>
<td>(R = 8.31)</td>
</tr>
<tr>
<td>Diatomic</td>
<td>Ideal</td>
<td>(\frac{7}{2}R = 29.10)</td>
<td>(\frac{5}{2}R = 20.79)</td>
<td>(R = 8.31)</td>
</tr>
<tr>
<td>Polyatomic</td>
<td>Ideal</td>
<td>(4R = 33.26)</td>
<td>(3R = 24.04)</td>
<td>(R = 8.31)</td>
</tr>
</tbody>
</table>

**Glossary**

**molar heat capacity at constant pressure**
quantifies the ratio of the amount of heat added removed to the temperature while measuring at constant pressure.

**molar heat capacity at constant volume**
quantifies the ratio of the amount of heat added removed to the temperature while measuring at constant volume.

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