3.6: Heat Capacities of an Ideal Gas

Learning Objectives

By the end of this section, you will be able to:

- Define heat capacity of an ideal gas for a specific process
- Calculate the specific heat of an ideal gas for either an isobaric or isochoric process
- Explain the difference between the heat capacities of an ideal gas and a real gas
- Estimate the change in specific heat of a gas over temperature ranges

We learned about specific heat and molar heat capacity previously; however, we have not considered a process in which heat is added. We do that in this section. First, we examine a process where the system has a constant volume, then contrast it with a system at constant pressure and show how their specific heats are related.

Let’s start with looking at Figure 1, which shows two vessels A and B, each containing 1 mol of the same type of ideal gas at a temperature T and a volume V. The only difference between the two vessels is that the piston at the top of A is fixed, whereas the one at the top of B is free to move against a constant external pressure p. We now consider what happens when the temperature of the gas in each vessel is slowly increased to (T + dT) with the addition of heat.
Figure \(\PageIndex{1}\): Two vessels are identical except that the piston at the top of A is fixed, whereas that atop B is free to move against a constant external pressure \(p\).

Since the piston of vessel A is fixed, the volume of the enclosed gas does not change. Consequently, the gas does no work, and we have from the first law

\[
\begin{align*}
\text{d}E_{\text{int}} &= \text{d}Q - \text{d}W = \text{d}Q. \\
\end{align*}
\]

We represent the fact that the heat is exchanged at constant volume by writing

\[
\begin{align*}
\text{d}Q &= C_V n \text{d}T, \\
\end{align*}
\]

where \(C_V\) is the molar heat capacity at constant volume of the gas. In addition, since \(\text{d}E_{\text{int}} = \text{d}Q\) for this particular process,

\[
\begin{align*}
\text{d}E_{\text{int}} &= C_V n \text{d}T. \\
\end{align*}
\]

We obtained this equation assuming the volume of the gas was fixed. However, internal energy is a state function that depends on only the temperature of an ideal gas. Therefore, \(\text{d}E_{\text{int}} = C_V n \text{d}T\) gives the change in internal energy of an ideal gas for any process involving a temperature change \(\text{d}T\).

When the gas in vessel B is heated, it expands against the movable piston and does work \(\text{d}W = pdV\). In this case, the heat is added at constant pressure, and we write \(\text{d}Q = C_p n \text{d}T\) where \(C_p\) is the molar heat capacity at constant pressure of the gas. Furthermore, since the ideal gas expands against a constant pressure,

\[
\begin{align*}
\text{d}(pV) &= \text{d}(RnT) \\
\end{align*}
\]

becomes \(pdV = Rn \text{d}T\). Finally, inserting the expressions for \(\text{d}Q\) and \(pdV\) into the first law, we obtain

\[
\begin{align*}
\text{d}E_{\text{int}} &= \text{d}Q - pdV = (C_p n - Rn) \text{d}T. \\
\end{align*}
\]

We have found \(\text{d}E_{\text{int}}\) for both an isochoric and an isobaric process. Because the internal energy of an ideal gas depends only on the temperature, \(\text{d}E_{\text{int}}\) must be the same for both processes. Thus,

\[
\begin{align*}
(C_V n) \text{d}T &= (C_p n - Rn) \text{d}T. \\
\end{align*}
\]
and

\[ C_p = C_V + R. \label{eq50} \]

The derivation of Equation \ref{eq50} was based only on the ideal gas law. Consequently, this relationship is approximately valid for all dilute gases, whether monatomic like He, diatomic like \((O_2)\), or polyatomic like \((CO_2)\) or \((NH_3)\).

In the preceding chapter, we found the molar heat capacity of an ideal gas under constant volume to be

\[ C_V = \dfrac{d}{2}R, \]

where \(d\) is the number of degrees of freedom of a molecule in the system. Table \(\PageIndex{1}\) shows the molar heat capacities of some dilute ideal gases at room temperature. The heat capacities of real gases are somewhat higher than those predicted by the expressions of \((C_V)\) and \((C_p)\) given in Equation \ref{eq50}. This indicates that vibrational motion in polyatomic molecules is significant, even at room temperature. Nevertheless, the difference in the molar heat capacities, \((C_p - C_V)\), is very close to \(R\), even for the polyatomic gases.

<table>
<thead>
<tr>
<th>Type of Molecule</th>
<th>Gas</th>
<th>((C_p)) (J/mol K)</th>
<th>((C_V)) (J/mol K)</th>
<th>((C_p - C_V)) (J/mol K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monatomic</td>
<td>Ideal</td>
<td>(\frac{5}{2}R = 20.79)</td>
<td>(\frac{3}{2}R = 12.47)</td>
<td>(R = 8.31)</td>
</tr>
<tr>
<td>Diatomic</td>
<td>Ideal</td>
<td>(\frac{7}{2}R = 29.10)</td>
<td>(\frac{5}{2}R = 20.79)</td>
<td>(R = 8.31)</td>
</tr>
<tr>
<td>Polyatomic</td>
<td>Ideal</td>
<td>(4R = 33.26)</td>
<td>(3R = 24.04)</td>
<td>(R = 8.31)</td>
</tr>
</tbody>
</table>

**Glossary**

**molar heat capacity at constant pressure**
quantifies the ratio of the amount of heat added removed to the temperature while measuring at constant pressure

**molar heat capacity at constant volume**
quantifies the ratio of the amount of heat added removed to the temperature while measuring at constant volume

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