15.6: The Magnetic Equivalent of Poisson's Equation

This deals with a static magnetic field, where there is no electrostatic field or at least any electrostatic field is indeed static – i.e. not changing. In that case \( \text{curl}\, \textbf{H} = \textbf{J} \). Now the magnetic field can be derived from the curl of the magnetic vector potential, defined by the two equations

\[
\textbf{B} = \textbf{curl}\, \textbf{A} \tag{15.6.1} \label{15.6.1}
\]

and

\[
\text{div}\, \textbf{A} = 0 . \tag{15.6.2} \label{15.6.2}
\]

(See Chapter 9 for a reminder of this.) Together with \( \textbf{H} = \textbf{B} / \mu \) (\( \mu \) = permeability), this gives us

\[
\textbf{curl}\, \textbf{curl}\, \textbf{A} = \mu \textbf{J} . \tag{15.6.3} \label{15.6.3}
\]

If we now remind ourselves of the jabberwockian-sounding vector differential operator equivalence

\[
\textbf{curl}\, \textbf{curl}\, \equiv \text{grad} \, \text{div} - \nabla^2 , \tag{15.6.4} \label{15.6.4}
\]

together with Equation \( \text{ref}{15.6.2} \), this gives us

\[
\nabla^2 \textbf{A} = -\mu \textbf{J} . \tag{15.6.5} \label{15.6.5}
\]

I don't know if this equation has any particular name, but it plays the same role for static magnetic fields that Poisson's equation plays for electrostatic fields. No matter what the distribution of currents, the magnetic vector potential at any point must obey Equation \( \text{ref}{15.6.5} \).
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