7.5: Magnetic Moment of a Plane, Current-carrying Coil

A plane, current carrying coil also experiences a torque in an external magnetic field, and its behaviour in a magnetic field is quite similar to that of a bar magnet or a compass needle. The torque is maximum when the normal to the coil is perpendicular to the magnetic field, and the magnetic moment is defined in exactly the same way, namely the maximum torque experienced in unit magnetic field.

Let us examine the behavior of a rectangular coil of sides \(a\) and \(b\), which is free to rotate about the dashed line shown in Figure \(\text{VII.2}\).

\[
\begin{array}{c}
\text{FIGURE VII.2} \\
\end{array}
\]

In Figure \(\text{VII.3}\), I am looking down the axis represented by the dashed line in Figure \(\text{VII.2}\), and we see the coil sideways on. A current \(I\) is flowing around the coil in the directions indicated by the symbols \(\bigodot\) and \(\bigotimes\). The normal to the coil makes an angle \(\theta\) with respect to an external field \(B\).
According to the Biot-Savart law there is a force \( \vec{F} \) on each of the \( b \)-length arms of magnitude \( bIB \), or, if there are \( N \) turns in the coil, \( \vec{F} = NbIB \). These two forces are opposite in direction and constitute a couple. The perpendicular distance between the two forces is \( a \sin \theta \), so the torque \( \tau \) on the coil is \( NabIB \sin \theta \), or \( \tau = NAIB \sin \theta \), where \( A \) is the area of the coil. This has its greatest value when \( \theta = 90^\circ \), and so the magnetic moment of the coil is \( NIA \).

This shows that, in SI units, magnetic moment can equally well be expressed in units of \( \text{N m T}^{-1} \), or ampere metre squared, which is dimensionally entirely equivalent to \( \text{A m}^2 \). Thus we have

\[
\tau = p_mB \sin \theta,
\]

where, for a plane current-carrying coil, the magnetic moment is

\[
p_m = NIA.
\]

This can conveniently be written in vector form:

\[
\tau = p_m \times \vec{B},
\]

where, for a plane current-carrying coil,

\[
p_m = N\vec{I}A.
\]

Here \( \vec{A} \) is a vector normal to the plane of the coil, with the current flowing clockwise around it. The vector \( \vec{B} \) is directed into the plane of the paper in Figure VII.3.

The formula \( NIA \) for the magnetic moment of a plane current-carrying coil is not restricted to rectangular coils, but holds equally for plane coils of any shape; for (see Figure VII.4) any curve can be described in terms of an infinite number of infinitesimally small vertical and horizontal segments.
We understand that a magnet, or anything that has a magnetic moment, experiences no net force in a uniform magnetic field, although it does experience a torque. Furthermore, as in the case of an electric dipole in an electric field, a magnetic dipole situated in an *inhomogeneous* magnetic field does experience a net force. If the magnetic moment and the gradient of the magnetic field are in the same direction, the net force on the dipole is

\[
p_m \frac{dB}{dx}.
\]

\[\text{N m T}^{-1} \times \text{T m}^{-1} = \text{N}.\]

See Section 3.5 for further details relating to a dipole in an inhomogeneous field.

An important historical experiment that readers may come across, using the force on a magnetic dipole in an inhomogeneous magnetic field, is the 1922 experiment of Stern and Gerlach, demonstrating the spatial quantization of the magnetic moment associated with electron spin.

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