14.S: Fluid Mechanics (Summary)

Key Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
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<tbody>
<tr>
<td>absolute pressure</td>
<td>sum of gauge pressure and atmospheric pressure</td>
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<tr>
<td>Archimedes’ principle</td>
<td>buoyant force on an object equals the weight of the fluid it displaces</td>
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<tr>
<td>Bernoulli’s equation</td>
<td>equation resulting from applying conservation of energy to an incompressible frictionless fluid: $p + \frac{1}{2} \rho v^2 + \rho gh = \text{constant}$ throughout the fluid</td>
</tr>
<tr>
<td>Bernoulli’s principle</td>
<td>Bernoulli’s equation applied at constant depth: $p_1 + \frac{1}{2} \rho v_1^2 = p_2 + \frac{1}{2} \rho v_2^2$</td>
</tr>
<tr>
<td>buoyant force</td>
<td>net upward force on any object in any fluid due to the pressure difference at different depths</td>
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<td>density</td>
<td>mass per unit volume of a substance or object</td>
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<tr>
<td>flow rate</td>
<td>abbreviated Q, it is the volume V that flows past a particular point during a time t, or $Q = \int \frac{dV}{dt}$</td>
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<tr>
<td>fluids</td>
<td>liquids and gases; a fluid is a state of matter that yields to shearing forces</td>
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<tr>
<td>gauge pressure</td>
<td>pressure relative to atmospheric pressure</td>
</tr>
<tr>
<td>hydraulic jack</td>
<td>simple machine that uses cylinders of different diameters to distribute force</td>
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</tbody>
</table>
hydrostatic equilibrium state at which water is not flowing, or is static

ideal fluid fluid with negligible viscosity

laminar flow type of fluid flow in which layers do not mix

Pascal's principle change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container

Poiseuille's law rate of laminar flow of an incompressible fluid in a tube: 

\[ Q = \frac{(p_{2} - p_{1})\pi r^{4}}{8\eta l} \]

Poiseuille's law for resistance resistance to laminar flow of an incompressible fluid in a tube: 

\[ R = \frac{8\eta l}{\pi r^{4}} \]

pressure force per unit area exerted perpendicular to the area over which the force acts

Reynolds number dimensionless parameter that can reveal whether a particular flow is laminar or turbulent

specific gravity ratio of the density of an object to a fluid (usually water)

turbulence fluid flow in which layers mix together via eddies and swirls

turbulent flow type of fluid flow in which layers mix together via eddies and swirls

viscosity measure of the internal friction in a fluid

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**Key Equations**

- Density of a sample at constant density
  \[ \rho = \frac{m}{V} \]

- Pressure
  \[ p = \frac{F}{A} \]

- Pressure at a depth h in a fluid of constant density
  \[ p = p_{0} + \rho gh \]

- Change of pressure with height in a constant-density fluid
  \[ \frac{dp}{dy} = -\rho g \]

- Absolute pressure
  \[ p_{\text{abs}} = p_{\text{g}} + p_{\text{atm}} \]

- Pascal's principle
  \[ \frac{F_{\text{1}}}{A_{\text{1}}} = \frac{F_{\text{2}}}{A_{\text{2}}} \]

- Volume flow rate
  \[ Q = \frac{dV}{dt} \]

- Continuity equation (constant density)
  \[ A_{\text{1}}v_{\text{1}} = A_{\text{2}}v_{\text{2}} \]
Continuity equation (general form)  \[ \rho_1 A_1 v_1 = \rho_2 A_2 v_2 \]
Bernoulli's equation  \[ p + \frac{1}{2} \rho v^2 + \rho gy = \text{constant} \]
Viscosity  \[ \eta = \frac{FL}{vA} \]
Poiseuille's law for resistance  \[ R = \frac{8 \eta l}{\pi r^4} \]
Poiseuille's law  \[ Q = \frac{(p_2 - p_1) \pi r^4}{8 \eta l} \]

Summary

14.1 Fluids, Density, and Pressure

- A fluid is a state of matter that yields to sideways or shearing forces. Liquids and gases are both fluids. Fluid statics is the physics of stationary fluids.
- Density is the mass per unit volume of a substance or object, defined as \( \rho = \frac{m}{V} \). The SI unit of density is kg/m\(^3\).
- Pressure is the force per unit perpendicular area over which the force is applied, \( p = \frac{F}{A} \). The SI unit of pressure is the pascal: 1 Pa = 1 N/m\(^2\).
- Pressure due to the weight of a liquid of constant density is given by \( p = \rho gh \), where \( p \) is the pressure, \( h \) is the depth of the liquid, \( \rho \) is the density of the liquid, and \( g \) is the acceleration due to gravity.

14.2 Measuring Pressure

- Gauge pressure is the pressure relative to atmospheric pressure.
- Absolute pressure is the sum of gauge pressure and atmospheric pressure.
- Open-tube manometers have U-shaped tubes and one end is always open. They are used to measure pressure. A mercury barometer is a device that measures atmospheric pressure.
- The SI unit of pressure is the pascal (Pa), but several other units are commonly used.

14.3 Pascal's Principle and Hydraulics

- Pressure is force per unit area.
- A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.
- A hydraulic system is an enclosed fluid system used to exert forces.
14.4 Archimedes’ Principle and Buoyancy

• Buoyant force is the net upward force on any object in any fluid. If the buoyant force is greater than the object’s weight, the object will rise to the surface and float. If the buoyant force is less than the object’s weight, the object will sink. If the buoyant force equals the object’s weight, the object can remain suspended at its present depth. The buoyant force is always present and acting on any object immersed either partially or entirely in a fluid.

• Archimedes’ principle states that the buoyant force on an object equals the weight of the fluid it displaces.

14.5 Fluid Dynamics

• Flow rate $Q$ is defined as the volume $V$ flowing past a point in time $t$, or $Q = \frac{dV}{dt}$ where $V$ is volume and $t$ is time. The SI unit of flow rate is m$^3$/s, but other rates can be used, such as L/min.

• Flow rate and velocity are related by $Q = Av$ where $A$ is the cross-sectional area of the flow and $v$ is its average velocity.

• The equation of continuity states that for an incompressible fluid, the mass flowing into a pipe must equal the mass flowing out of the pipe.

14.6 Bernoulli’s Equation

• Bernoulli’s equation states that the sum on each side of the following equation is constant, or the same at any two points in an incompressible frictionless fluid: $p_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2$.

• Bernoulli’s principle is Bernoulli’s equation applied to situations in which the height of the fluid is constant. The terms involving depth (or height $h$) subtract out, yielding $p_1 + \frac{1}{2} \rho v_1^2 = p_2 + \frac{1}{2} \rho v_2^2$.

• Bernoulli’s principle has many applications, including entrainment and velocity measurement.

14.7 Viscosity and Turbulence

• Laminar flow is characterized by smooth flow of the fluid in layers that do not mix.

• Turbulence is characterized by eddies and swirls that mix layers of fluid together.

• Fluid viscosity $\eta$ is due to friction within a fluid.

• Flow is proportional to pressure difference and inversely proportional to resistance: $Q = \frac{p - p_1}{R}$.

• The pressure drop caused by flow and resistance is given by $p_2 - p_1 = RQ$.

• The Reynolds number $N_R$ can reveal whether flow is laminar or turbulent. It is $N_R = \frac{\rho vr}{\eta}$.

• For $N_R$ below about 2000, flow is laminar. For $N_R$ above about 3000, flow is turbulent. For values of $N_R$ between 2000 and 3000, it may be either or both.