### 14.1: Introduction to General Perturbation Theory

A particle in orbit around a point mass – or a spherically symmetric mass distribution – is moving in a gravitational potential of the form \(-\frac{GM}{r}\). In this potential it moves in a keplerian ellipse (or hyperbola if its kinetic energy is large enough) that can be described by the six orbital elements \(a, e, i, \Omega, \omega, T\), or any equivalent set of six parameters.

If the potential is a little different from \(-\frac{GM}{r}\), say \(-\frac{GM}{r + R}\), the orbit will be perturbed, and \(R\) is described as a perturbation. As a result it will no longer move in a perfect keplerian ellipse. Perturbations may be periodic or secular. For example, the elements such as \(a, e\) or \(i\) may vary in a periodic fashion, while there may be secular changes (i.e. changes that are not periodic but constantly increase or decrease in the same direction) in elements such as \(\Omega\) and \(\omega\). (That is, the line of nodes and the line of apsides may monotonically precess; they may advance or regress.)

In some situations it may be possible to express the perturbation in terms of a simple algebraic formula. An example would be a particle in orbit around a slightly oblate planet, where it is possible to express the potential algebraically. The aim of this chapter will be to try to find general expressions for the rates of change of the orbital elements in terms of the perturbing function, and we shall use the orbit around an oblate planet as an example.

In other situations it is not easily possible to express the perturbation in terms of a simple algebraic function. For example, a planet in orbit around the Sun is subject not only to the gravitational field of the Sun, but to the perturbations caused by all the other planets in the solar system. These special perturbations have to be treated numerically, and the techniques for doing so will be described in chapter 15.

- Jeremy Tatum (University of Victoria, Canada)