1.2: Plane Triangular Lamina

Definition: A median of a triangle is a line from a vertex to the midpoint of the opposite side.

Theorem I. The three medians of a triangle are concurrent (meet at a single, unique point) at a point that is two-thirds of the distance from a vertex to the midpoint of the opposite side.

Theorem II. The centre of mass of a uniform triangular lamina (or the centroid of a triangle) is at the meet of the medians.

The proof of I can be done with a nice vector argument (Figure I.1):

Let \( \textbf{A}, \textbf{B} \) be the vectors \( \text{OA}, \text{OB} \). Then \( \textbf{A+B} \) is the diagonal of the parallelogram of which \( \text{OA} \) and \( \text{OB} \) are two sides, and the position vector of the point \( \text{C}_1 \) is \( \frac{1}{3}(\textbf{A+B}) \).

To get \( \text{C}_2 \), we see that

\[
\text{C}_2 = \text{A} + \frac{2}{3}(\text{AM}_2) = \text{A} + \frac{2}{3}(\frac{1}{2}\text{B-A}) = \frac{1}{3}(\textbf{A+B})
\]
Thus the points \(C_1\) and \(C_2\) are identical, and the same would be true for the third median, so Theorem I is proved.

Now consider an elemental slice as in Figure I.2. The centre of mass of the slice is at its mid-point. The same is true of any similar slices parallel to it. Therefore the centre of mass is on the locus of the mid-points - i.e. on a median. Similarly, it is on each of the other medians, and Theorem II is proved.

![Figure I.2](image-url)

That needed only some vector geometry. We now move on to some calculus.

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