1.2: Plane Triangular Lamina

Definition: A median of a triangle is a line from a vertex to the midpoint of the opposite side.

Theorem I. The three medians of a triangle are concurrent (meet at a single, unique point) at a point that is two-thirds of the distance from a vertex to the midpoint of the opposite side.

Theorem II. The centre of mass of a uniform triangular lamina (or the centroid of a triangle) is at the meet of the medians.

The proof of I can be done with a nice vector argument (Figure 1.1):

Let $\bf{A}$, $\bf{B}$ be the vectors $\text{OA}$, $\text{OB}$. Then $\bf{A+B}$ is the diagonal of the parallelogram of which $\text{OA}$ and $\text{OB}$ are two sides, and the position vector of the point $\text{C}_{1}$ is $\frac{1}{3}(\bf{A+B})$.

To get $\text{C}_{2}$, we see that

$$\text{(1)}$$

$\text{C}_{2} = \bf{A} + \frac{2}{3}(\text{AM}_{2}) = \bf{A} + \frac{2}{3}(\frac{1}{2}\bf{B- A}) = \frac{1}{3}(\bf{A+B})$
Thus the points \((\text{C}_1, 1)\) and \((\text{C}_2, 2)\) are identical, and the same would be true for the third median, so Theorem I is proved.

Now consider an elemental slice as in Figure 1.2. The centre of mass of the slice is at its mid-point. The same is true of any similar slices parallel to it. Therefore the centre of mass is on the locus of the mid-points - i.e. on a median. Similarly, it is on each of the other medians, and Theorem II is proved.

![Figure 1.2](image)

That needed only some vector geometry. We now move on to some calculus.

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