5.2: Bouncing Balls

When a ball is dropped to the ground, one of four things may happen:

1. It may rebound with exactly the same speed as the speed at which it hit the ground. This is an *elastic collision*.
2. It may come to a complete rest, for example if it were a ball of soft putty. I shall call this a completely *inelastic collision*.
3. It may bounce back, but with a reduced speed. For want of a better term I shall refer to this as a somewhat *inelastic collision*.
4. If there happens to be a little heap of gunpowder lying on the table where the ball hits it, it may bounce back with a faster speed than it had immediately before collision. That would be a *superelastic collision*.

The ratio

$$\dfrac{\text{speed after collision}}{\text{speed before collision}} \nonumber$$

is called the *coefficient of restitution*, for which I shall use the speed before collision symbol e. The coefficient is 1 for an elastic collision, less than 1 for an inelastic collision, zero for a completely inelastic collision, and greater than 1 for a **superelastic** collision. The ratio of kinetic energy (after) to kinetic energy (before) is evidently, *in this situation*, e^{2}.

If a ball falls on to a table from a height h_{0}, it will take a time $t_{0} = \sqrt{2H_{0}lg}$ to fall. If the collision is somewhat inelastic it will then rise to a height $h_{1}=e^{2}h_{0}$ and it will take a time et to reach height h_{1}. Then it will fall again, and bounce again, this time to a lesser height. And, if the coefficient of restitution remains the same, it will continue to do this for an infinite number of bounces. After a billion bounces, there is still an infinite number of bounces yet to come. The total distance travelled is

$$h = h_{0} +2h_{0}(e^{2}+e^{4}+e^{6}+...) \tag{5.2.1}\label{eq:5.2.1}$$

and the time taken is

$$ t = t_{0} +2t_{0}(e + e^{2}+e^{3}+...). \tag{5.2.2}\label{eq:5.2.2} $$

These are geometric series, and their sums are

$$ h = h_{0} \left(\frac{1+e^{2}}{1-e^{2}}\right), \tag{5.2.3}\label{eq:5.2.3} $$

which is independent of g (i.e. of the planet on which this experiment is performed), and

$$ t = t_{0} \left(\frac{1+e}{1-e} \right) \tag{5.2.4}\label{eq:5.2.4} $$

For example, suppose h_{0} = 1 m, e = 0.5, g = 9.8 m s^{-2}, then the ball comes to rest in 1.36 s after having travelled 1.67 m after an infinite number of bounces.

Discuss

Does the ball ever stop bouncing, given that, after every bounce, there is still an infinite number yet to come; yet after 1.36 seconds it is no longer bouncing...?