5.2: Bouncing Balls

When a ball is dropped to the ground, one of four things may happen:

1. It may rebound with exactly the same speed as the speed at which it hit the ground. This is an **elastic collision**.
2. It may come to a complete rest, for example if it were a ball of soft putty. I shall call this a completely **inelastic collision**.
3. It may bounce back, but with a reduced speed. For want of a better term I shall refer to this as a somewhat **inelastic collision**.
4. If there happens to be a little heap of gunpowder lying on the table where the ball hits it, it may bounce back with a faster speed than it had immediately before collision. That would be a **superelastic collision**.

The ratio

\[
\frac{\text{speed after collision}}{\text{speed before collision}}
\]

is called the **coefficient of restitution**, for which I shall use the speed before collision symbol \(e\). The coefficient is 1 for an elastic collision, less than 1 for an inelastic collision, zero for a completely inelastic collision, and greater than 1 for a superelastic collision. The ratio of kinetic energy (after) to kinetic energy (before) is evidently, in this situation, \(e^2\).

If a ball falls on to a table from a height \(h_0\), it will take a time \(t_0 = \sqrt{2H_0/\rho g}\) to fall. If the collision is somewhat inelastic it will then rise to a height \(h_1 = e^2 h_0\) and it will take a time \(et\) to reach height \(h_1\). Then it will fall again, and bounce again, this time to a lesser height. And, if the coefficient of restitution remains the same, it will continue to do this for an infinite number of bounces. After a billion bounces, there is still an infinite number of bounces yet to come. The total distance travelled is

\[
h = h_0 + 2h_0(e^2 + e^4 + e^6 + \ldots) \tag{5.2.1}\]

\[\text{label eq:5.2.1}\]
and the time taken is

\[ t = t_{0} + 2t_{0}(e + e^2 + e^3 + ...). \tag{5.2.2} \label{eq:5.2.2} \]

These are geometric series, and their sums are

\[ h = h_{0} \left(\frac{1+e^2}{1-e^2}\right), \tag{5.2.3} \label{eq:5.2.3} \]

which is independent of g (i.e. of the planet on which this experiment is performed), and

\[ t = t_{0} \left(\frac{1+e}{1-e}\right) \tag{5.2.4} \label{eq:5.2.4} \]

For example, suppose \(( h_{0} = 1 \text{ m}, \; e = 0.5, \; g = 9.8 \text{ m s}^{-2}\)), then the ball comes to rest in 1.36 s after having travelled 1.67 m after an infinite number of bounces.

Discuss

Does the ball ever stop bouncing, given that, after every bounce, there is still an infinite number yet to come; yet after 1.36 seconds it is no longer bouncing...

---

**Contributor**

- Jeremy Tatum (University of Victoria, Canada)