1.14: Relations between Flux, Intensity, Exitance, Irradiance

In this section I am going to ask, and answer, three questions.

i. (See figure I.3 )

A point source of light has an intensity that varies with direction as \( I(\theta, \phi) \). What is the radiant flux radiated into the hemisphere \( \theta < \frac{\pi}{2} \)? This is easy; we already answered it for a complete sphere in equation 1.6.3. For a hemisphere, the answer is

\[
\phi = \int_0^{2\pi} \int_0^{\pi/2} I(\theta, \phi) \sin \theta \, d\theta \, d\phi. \tag{1.14.1} \label{1.14.1}
\]

ii. At a certain point on an extended plane radiating surface, the radiance is \( L(\theta, \phi) \). What is the emergent exitance \( M \) at that point?
Consider an elemental area $\delta A$ (see figure I.4). The intensity $I(\theta, \phi)$ radiated in the direction $(\theta, \phi)$ is the radiance times the projected area $\cos \theta \ \delta A$. Therefore the radiant power or flux radiated by the element into the hemisphere is

$$\delta \phi = \int_0^{2\pi} \int_0^{\pi/2} L(\theta, \phi) \cos \theta \sin \theta \ d\theta \ d\phi \ \delta A, \tag{1.14.2}$$

and therefore the exitance is

$$M = \int_0^{2\pi} \int_0^{\pi/2} L(\theta, \phi) \cos \theta \sin \theta \ d\theta \ d\phi \ \tag{1.14.3} \label{1.14.3}$$

iii. A point $O$ is at the centre of the base of a hollow radiating hemisphere whose radiance in the direction $(\theta, \phi)$ is $L(\theta, \phi)$. What is the irradiance at that point $O$? (See figure I.5.)

Consider an elemental area $(a^2 \ \sin \theta \ \delta \theta \ \delta \phi)$ on the inside of the hemisphere at a point where the radiance is $L(\theta, \phi)$ (figure I.5). The intensity radiated towards $(O)$ is the radiance times the area:
The irradiance at \(O\) from this elemental area is (see equation (1.10.1))

\[
\delta E = \frac{\delta I (\theta, \phi) \cos \theta}{a^2} = L (\theta, \phi) \cos \theta \sin \theta \delta \theta \delta \phi,
\]

\tag{1.14.5} \label{1.14.5}

and so the irradiance at \(O\) from the entire hemisphere is

\[
E = \int_0^{2\pi} \int_0^{\pi/2} L(\theta, \phi) \cos \theta \sin \theta \delta \theta \delta \phi.
\]

\tag{1.14.6} \label{1.14.6}

The same would apply for any shape of inverted bowl - or even an infinite plane radiating surface (see figure I.6.)

\text{(FIGURE I.6)}

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