3.2: Newton's Laws of motion

Newton defined a vector quantity called linear momentum $\mathbf{p}$ which is the product of mass and velocity.

\begin{equation}
\mathbf{p} = m\dot{\mathbf{r}}
\end{equation}

Since the mass $(m)$ is a scalar quantity, then the velocity vector $(\dot{r})$ and the linear momentum vector $(\mathbf{p})$ are colinear.

Newton’s laws, expressed in terms of linear momentum, are:

1. **Law of inertia**: A body remains at rest or in uniform motion unless acted upon by a force.
2. **Equation of motion**: A body acted upon by a force moves in such a manner that the time rate of change of momentum equals the force.\begin{equation}
\mathbf{F} = \frac{d\mathbf{p}}{dt}
\end{equation}
3. **Action and reaction**: If two bodies exert forces on each other these forces are equal in magnitude and opposite in direction.

Newton’s second law contains the essential physics relating the force $(\mathbf{F})$ and the rate of change of linear momentum $(\mathbf{p})$.

Newton’s first law, the law of inertia, is a special case of Newton’s second law in that if

\begin{equation}
\mathbf{F} = \frac{d\mathbf{p}}{dt} = 0
\end{equation}

then $(\mathbf{p})$ is a constant of motion.
Newton’s third law also can be interpreted as a statement of the conservation of momentum, that is, for a two particle system with no external forces acting,

\begin{equation} \label{eq:2.4} \mathbf{F}_{12} = -\mathbf{F}_{21} \end{equation}

If the forces acting on two bodies are their mutual action and reaction, then Equation \ref{eq:2.4} simplifies to

\begin{equation} \label{eq:2.5} 
\frac{d}{dt}(\mathbf{p_1}+\mathbf{p_2}) = 0 
\end{equation}

This implies that the total linear momentum \(\mathbf{P = p_1 + p_2}\) is a constant of motion. Combining Equations \ref{eq:2.1} and \ref{eq:2.2} leads to a second-order differential equation

\begin{equation} \label{2.6} \mathbf{F} = \frac{d\mathbf{p}}{dt} = m\frac{d^2\mathbf{r}}{dt^2} = m\mathbf{\ddot{r}} \end{equation}

Note that the force on a body \(\mathbf{F}\), and the resultant acceleration \((\mathbf{a = \ddot{r}})\) are colinear. Appendix \(\text{(C2)}\) gives explicit expressions for the acceleration \((\mathbf{a})\) in cartesian and curvilinear coordinate systems. The definition of force depends on the definition of the mass \((m)\). Newton’s laws of motion are obeyed to a high precision for velocities much less than the velocity of light. For example, recent experiments have shown they are obeyed with an error in the acceleration of \(\Delta a \leq 5 \times 10^{-14}\text{m/s}^2\).