9.1 Current and Voltage

9.1.1 Current

Unity of all types of electricity

We are surrounded by things we have been told are “electrical,” but it's far from obvious what they have in common to justify being grouped together. What relationship is there between the way socks cling together and the way a battery lights a lightbulb? We have been told that both an electric eel and our own brains are somehow electrical in nature, but what do they have in common?

a / Gymnotus carapo, a knifefish, uses electrical signals to sense its environment and to communicate with others of its species. (Greg DeGreef)
British physicist Michael Faraday (1791-1867) set out to address this problem. He investigated electricity from a variety of sources --- including electric eels! --- to see whether they could all produce the same effects, such as shocks and sparks, attraction and repulsion. “Heating” refers, for example, to the way a lightbulb filament gets hot enough to glow and emit light. Magnetic induction is an effect discovered by Faraday himself that connects electricity and magnetism. We will not study this effect, which is the basis for the electric generator, in detail until later in the book.

The table shows a summary of some of Faraday's results. Check marks indicate that Faraday or his close contemporaries were able to verify that a particular source of electricity was capable of producing a certain effect. (They evidently failed to demonstrate attraction and repulsion between objects charged by electric eels, although modern workers have studied these species in detail and been able to understand all their electrical characteristics on the same footing as other forms of electricity.)

Faraday's results indicate that there is nothing fundamentally different about the types of electricity supplied by the various sources. They are all able to produce a wide variety of identical effects. Wrote Faraday, “The general conclusion which must be drawn from this collection of facts is that electricity, whatever may be its source, is identical in its nature.”

If the types of electricity are the same thing, what thing is that? The answer is provided by the fact that all the sources of electricity can cause objects to repel or attract each other. We use the word “charge” to describe the property of an object that allows it to participate in such electrical forces, and we have learned that charge is present in matter in the form of nuclei and electrons. Evidently all these electrical phenomena boil down to the motion of charged particles in matter.

---

**Electric current**

If the fundamental phenomenon is the motion of charged particles, then how can we define a useful numerical measurement of it? We might describe the flow of a river simply by the velocity of the water, but velocity will not be appropriate for electrical purposes because we need to take into account how much charge the moving particles have, and in any case there are no practical devices sold at Radio Shack that can tell us the velocity of charged particles. Experiments show that the intensity of various electrical effects is related to a different quantity: the number of coulombs of charge that pass by a certain point per second. By analogy with the flow of water, this quantity is called the electric current, \(I\). Its units of coulombs/second are more conveniently abbreviated as amperes, \(1\ A=1\ C/s\). (In informal speech, one usually says “amps.”)
The main subtlety involved in this definition is how to account for the two types of charge. The stream of water coming from a hose is made of atoms containing charged particles, but it produces none of the effects we associate with electric currents. For example, you do not get an electrical shock when you are sprayed by a hose. This type of experiment shows that the effect created by the motion of one type of charged particle can be canceled out by the motion of the opposite type of charge in the same direction. In water, every oxygen atom with a charge of \((+8e)\) is surrounded by eight electrons with charges of \((-e)\), and likewise for the hydrogen atoms.

We therefore refine our definition of current as follows:

When charged particles are exchanged between regions of space A and B, the electric current flowing from A to B is defined as

\[
I = \frac{dq}{dt},
\]

where \((dq)\) is the change in region B's total charge occurring over a period of time \((dt)\).

In the garden hose example, your body picks up equal amounts of positive and negative charge, resulting in no change in your total charge, so the electrical current flowing into you is zero.

Example 1: Ions moving across a cell membrane

\(\triangleright\) Figure b shows ions, labeled with their charges, moving in or out through the membranes of four cells. If the ions all cross the membranes during the same interval of time, how would the currents into the cells compare with each other?

\(\triangleright\) We're just assuming the rate of flow is constant, so we can talk about \((\Delta q)\) instead of \((dq)\).

Cell A has positive current going into it because its charge is increased, i.e., has a positive value of \((\Delta q)\).

Cell B has the same current as cell A, because by losing one unit of negative charge it also ends up increasing its own total charge by one unit.

Cell C's total charge is reduced by three units, so it has a large negative current going into it.

Cell D loses one unit of charge, so it has a small negative current into it.
**Example 2: Finding current given charge**

A charged balloon falls to the ground, and its charge begins leaking off to the Earth. Suppose that the charge on the balloon is given by \( q = ae^{-bt} \). Find the current as a function of time, and interpret the answer.

Taking the derivative, we have

\[
\begin{align*}
I &= \frac{dq}{dt} \\
&= -abe^{-bt}
\end{align*}
\]

An exponential function approaches zero as the exponent gets more and more negative. This means that both the charge and the current are decreasing in magnitude with time. It makes sense that the charge approaches zero, since the balloon is losing its charge. It also makes sense that the current is decreasing in magnitude, since charge cannot flow at the same rate forever without overshooting zero.

The reverse of differentiation is integration, so if we know the current as a function of time, we can find the charge by integrating. Example 8 on page 522 shows such a calculation.

It may seem strange to say that a negatively charged particle going one way creates a current going the other way, but this is quite ordinary. As we will see, currents flow through metal wires via the motion of electrons, which are negatively charged, so the direction of motion of the electrons in a circuit is always opposite to the direction of the current. Of course it would have been convenient of Benjamin Franklin had defined the positive and negative signs of charge the opposite way, since so many electrical devices are based on metal wires.

**Example 3: Number of electrons flowing through a lightbulb**

If a lightbulb has 1.0 A flowing through it, how many electrons will pass through the filament in 1.0 s?

We are only calculating the number of electrons that flow, so we can ignore the positive and negative signs. Also, since the rate of flow is constant, we don't really need to think in terms of calculus; the derivative \( \frac{dq}{dt} \) that defines current is the same as \( \frac{\Delta q}{\Delta t} \) in this situation. Solving for \( \Delta q = I \Delta t \) gives a charge of 1.0 C flowing in this time interval. The number of electrons is

\[
\begin{align*}
\text{number of electrons} &= \frac{\text{coulombs}}{\text{coulomb}} \times \frac{\text{electrons}}{\text{coulomb}} \\
&= \frac{1.0}{6.2 \times 10^{18}}
\end{align*}
\]

How can we put electric currents to work? The only method of controlling electric charge we have studied so far is to charge different substances, e.g., rubber and fur, by rubbing them against each other. Figure c/1 shows an attempt to use this technique to light a lightbulb. This method is unsatisfactory. True, current will flow through the bulb, since electrons can move through metal wires, and the excess electrons on the rubber rod will therefore come through the wires and bulb due to the attraction of the positively charged fur and the repulsion of the other electrons. The problem is that after a zillionth of a second of current, the rod and fur will both have run out of charge. No more current will flow, and the lightbulb will go out.

Figure c/2 shows a setup that works. The battery pushes charge through the circuit, and recycles it over and over again. (We will have more to say later in this chapter about how batteries work.) This is called a complete circuit. Today, the electrical use of the word “circuit” is the only one that springs to mind for most people, but the original meaning was to travel around and make a round trip, as when a circuit court judge would ride around the boondocks, dispensing justice in each town on a certain date.

Note that an example like c/3 does not work. The wire will quickly begin acquiring a net charge, because it has no way to get rid of the charge flowing into it. The repulsion of this charge will make it more and more difficult to send any more charge in, and soon the electrical forces exerted by the battery will be canceled out completely. The whole process would be over so
quickly that the filament would not even have enough time to get hot and glow. This is known as an \textit{open circuit}. Exactly the same thing would happen if the complete circuit of figure \(c/2\) was cut somewhere with a pair of scissors, and in fact that is essentially how an ordinary light switch works: by opening up a gap in the circuit.

The definition of electric current we have developed has the great virtue that it is easy to measure. In practical electrical work, one almost always measures current, not charge. The instrument used to measure current is called an \textit{ammeter}. A simplified ammeter, \(c/4\), simply consists of a coiled-wire magnet whose force twists an iron needle against the resistance of a spring. The greater the current, the greater the force. Although the construction of ammeters may differ, their use is always the same. We break into the path of the electric current and interpose the meter like a tollbooth on a road, \(c/5\). There is still a complete circuit, and as far as the battery and bulb are concerned, the ammeter is just another segment of wire.

Does it matter where in the circuit we place the ammeter? Could we, for instance, have put it in the left side of the circuit instead of the right? Conservation of charge tells us that this can make no difference. Charge is not destroyed or “used up” by the lightbulb, so we will get the same current reading on either side of it. What is “used up” is energy stored in the battery, which is being converted into heat and light energy.

\section*{9.1.3 Voltage}

\subsection*{The volt unit}

Electrical circuits can be used for sending signals, storing information, or doing calculations, but their most common purpose by far is to manipulate energy, as in the battery-and-bulb example of the previous section. We know that lightbulbs are rated in units of watts, i.e., how many joules per second of energy they can convert into heat and light, but how would this relate to the flow of charge as measured in amperes? By way of analogy, suppose your friend, who didn't take physics, can't find any job better than pitching bales of hay. The number of calories he burns per hour will certainly depend on how many bales he pitches per minute, but it will also be proportional to how much mechanical work he has to do on each bale. If his job is to toss them up into a hayloft, he will get tired a lot more quickly than someone who merely tips bales off a loading dock into trucks. In metric units,

\begin{equation*}
\frac{\text{joules}}{\text{second}} = \frac{\text{haybales}}{\text{second}} \times \frac{\text{joules}}{\text{haybale}}.
\end{equation*}

Similarly, the rate of energy transformation by a battery will not just depend on how many coulombs per second it pushes through a circuit but also on how much mechanical work it has to do on each coulomb of charge:

\begin{equation*}
\frac{\text{joules}}{\text{second}} = \frac{\text{coulombs}}{\text{second}} \times \frac{\text{joules}}{\text{coulomb}}.
\end{equation*}

or

\begin{equation*}
\text{power} = \text{current} \times \text{work per unit charge}.
\end{equation*}

Units of joules per coulomb are abbreviated as \textit{volts}, 1 V=1 J/C, named after the Italian physicist Alessandro Volta. Everyone knows that batteries are rated in units of volts, but the voltage concept is more general than that; it turns out that voltage is a
property of every point in space. To gain more insight, let's think more carefully about what goes on in the battery and bulb circuit.

**The voltage concept in general**

To do work on a charged particle, the battery apparently must be exerting forces on it. How does it do this? Well, the only thing that can exert an electrical force on a charged particle is another charged particle. It's as though the haybales were pushing and pulling each other into the hayloft! This is potentially a horribly complicated situation. Even if we knew how much excess positive or negative charge there was at every point in the circuit (which realistically we don't) we would have to calculate zillions of forces using Coulomb's law, perform all the vector additions, and finally calculate how much work was being done on the charges as they moved along. To make things even more scary, there is more than one type of charged particle that moves: electrons are what move in the wires and the bulb's filament, but ions are the moving charge carriers inside the battery. Luckily, there are two ways in which we can simplify things:

**The situation is unchanging.** Unlike the imaginary setup in which we attempted to light a bulb using a rubber rod and a piece of fur, this circuit maintains itself in a steady state (after perhaps a microsecond-long period of settling down after the circuit is first assembled). The current is steady, and as charge flows out of any area of the circuit it is replaced by the same amount of charge flowing in. The amount of excess positive or negative charge in any part of the circuit therefore stays constant. Similarly, when we watch a river flowing, the water goes by but the river doesn't disappear.

**Force depends only on position.** Since the charge distribution is not changing, the total electrical force on a charged particle depends only on its own charge and on its location. If another charged particle of the same type visits the same location later on, it will feel exactly the same force.

The second observation tells us that there is nothing all that different about the experience of one charged particle as compared to another's. If we single out one particle to pay attention to, and figure out the amount of work done on it by electrical forces as it goes from point A to point B along a certain path, then this is the same amount of work that will be done on any other charged particles of the same type as it follows the same path. For the sake of visualization, let's think about the path that starts at one terminal of the battery, goes through the light bulb's filament, and ends at the other terminal. When an object experiences a force that depends only on its position (and when certain other, technical conditions are satisfied), we can define an electrical energy associated with the position of that object. The amount of work done on the particle by electrical forces as it moves from A to B equals the drop in electrical energy between A and B. This electrical energy is what is being converted into other forms of energy such as heat and light. We therefore define voltage in general as electrical energy per unit charge:

\[ \Delta V = \Delta U_{elec}/q \]

where \(\Delta U_{elec}\) is the change in the electrical energy of a particle with charge \(q\) as it moves from the initial point to the final point.

The amount of power dissipated (i.e., rate at which energy is transformed by the flow of electricity) is then given by the equation
\[ P = I \Delta V. \]

**Example 4: Energy stored in a battery**

\( \triangleright \) The 1.2 V rechargeable battery in figure d is labeled 1800 milliamp-hours. What is the maximum amount of energy the battery can store?

\( \triangleright \) An ampere-hour is a unit of current multiplied by a unit of time. Current is charge per unit time, so an ampere-hour is in fact a funny unit of charge:

\[
\begin{align*}
(1 \text{ A})(1 \text{ hour}) &= (1 \text{ C/s})(3600 \text{ s}) \\
&= 3600 \text{ C}
\end{align*}
\]

1800 milliamp-hours is therefore \( (1800 \times 10^{-3}) \times 3600 \text{ C} = 6.5 \times 10^3 \text{ C} \). That's a huge number of charged particles, but the total loss of electrical energy will just be their total charge multiplied by the voltage difference across which they move:

\[
\begin{align*}
\Delta U_{\text{elec}} &= q \Delta V \\
&= (6.5 \times 10^3 \text{ C})(1.2 \text{ V}) \\
&= 7.8 \text{ kJ}
\end{align*}
\]

**Example 5: Units of volt-amps**

\( \triangleright \) Doorbells are often rated in volt-amps. What does this combination of units mean?

\( \triangleright \) Current times voltage gives units of power, \( P = I \Delta V \), so volt-amps are really just a nonstandard way of writing watts. They are telling you how much power the doorbell requires.

**Example 6: Power dissipated by a battery and bulb**

\( \triangleright \) If a 9.0-volt battery causes 1.0 A to flow through a lightbulb, how much power is dissipated?

\( \triangleright \) The voltage rating of a battery tells us what voltage difference \( \Delta V \) it is designed to maintain between its terminals.
\[ P = I \Delta V = 9.0 \text{A} \cdot \text{V} = 9.0 \frac{\text{C}}{\text{s}} \cdot \frac{\text{J}}{\text{C}} = 9.0 \text{J/s} = 9.0 \text{W} \]

The only nontrivial thing in this problem was dealing with the units. One quickly gets used to translating common combinations like \((\text{A} \cdot \text{V})\) into simpler terms.

Here are a few questions and answers about the voltage concept.

**Question:** OK, so what *is* voltage, really?
**Answer:** A device like a battery has positive and negative charges inside it that push other charges around the outside circuit. A higher-voltage battery has denser charges in it, which will do more work on each charged particle that moves through the outside circuit.

To use a gravitational analogy, we can put a paddlewheel at the bottom of either a tall waterfall or a short one, but a kg of water that falls through the greater gravitational energy difference will have more energy to give up to the paddlewheel at the bottom.

**Question:** Why do we define voltage as electrical energy divided by charge, instead of just defining it as electrical energy?
**Answer:** One answer is that it’s the only definition that makes the equation \((P=I \Delta V)\) work. A more general answer is that we want to be able to define a voltage difference between any two points in space without having to know in advance how much charge the particles moving between them will have. If you put a nine-volt battery on your tongue, then the charged particles that move across your tongue and give you that tingly sensation are not electrons but ions, which may have charges of \((+e), (-2e),\) or practically anything. The manufacturer probably expected the battery to be used mostly in circuits with metal wires, where the charged particles that flowed would be electrons with charges of \((-e)\). If the ones flowing across your tongue happen to have charges of \((-2e)\), the electrical energy difference for them will be twice as much, but dividing by their charge of \((-2e)\) in the definition of voltage will still give a result of 9 V.

**Question:** Are there two separate roles for the charged particles in the circuit, a type that sits still and exerts the forces, and another that moves under the influence of those forces?
**Answer:** No. Every charged particle simultaneously plays both roles. Newton’s third law says that any particle that has an electrical force acting on it must also be exerting an electrical force back on the other particle. There are no “designated movers” or “designated force-makers.”

**Question:** Why does the definition of voltage only refer to voltage *differences*?
**Answer:** It’s perfectly OK to define voltage as \((V=U_{\text{elec}}/q)\). But recall that it is only *differences* in interaction energy, \((U)\), that have direct physical meaning in physics. Similarly, voltage differences are really more useful than absolute voltages. A voltmeter measures voltage differences, not absolute voltages.

**Discussion Questions**

◊ A roller coaster is sort of like an electric circuit, but it uses gravitational forces on the cars instead of electric ones. What would a high-voltage roller coaster be like? What would a high-current roller coaster be like?
Criticize the following statements:

- “He touched the wire, and 10000 volts went through him.”
- “That battery has a charge of 9 volts.”
- “You used up the charge of the battery.”

When you touch a 9-volt battery to your tongue, both positive and negative ions move through your saliva. Which ions go which way?

I once touched a piece of physics apparatus that had been wired incorrectly, and got a several-thousand-volt voltage difference across my hand. I was not injured. For what possible reason would the shock have had insufficient power to hurt me?

9.1.4 Resistance

Resistance

So far we have simply presented it as an observed fact that a battery-and-bulb circuit quickly settles down to a steady flow, but why should it? Newton's second law, \( a = \frac{F}{m} \), would seem to predict that the steady forces on the charged particles should make them whip around the circuit faster and faster. The answer is that as charged particles move through matter, there are always forces, analogous to frictional forces, that resist the motion. These forces need to be included in Newton's second law, which is really \( a = \frac{F_{\text{total}}}{m} \), not \( a = \frac{F}{m} \). If, by analogy, you push a crate across the floor at constant speed, i.e., with zero acceleration, the total force on it must be zero. After you get the crate going, the floor's frictional force is exactly canceling out your force. The chemical energy stored in your body is being transformed into heat in the crate and the floor, and no longer into an increase in the crate's kinetic energy. Similarly, the battery's internal chemical energy is converted into heat, not into perpetually increasing the charged particles' kinetic energy. Changing energy into heat may be a nuisance in some circuits, such as a computer chip, but it is vital in a lightbulb, which must get hot enough to glow. Whether we like it or not, this kind of heating effect is going to occur any time charged particles move through matter.

What determines the amount of heating? One flashlight bulb designed to work with a 9-volt battery might be labeled 1.0 watts, another 5.0. How does this work? Even without knowing the details of this type of friction at the atomic level, we can relate the heat dissipation to the amount of current that flows via the equation \( P = I \Delta V \). If the two flashlight bulbs can have two different values of \( P \) when used with a battery that maintains the same \( \Delta V \), it must be that the 5.0-watt bulb allows five times more current to flow through it.

For many substances, including the tungsten from which lightbulb filaments are made, experiments show that the amount of current that will flow through it is directly proportional to the voltage difference placed across it. For an object made of such a substance, we define its electrical resistance as follows:

If an object inserted in a circuit displays a current flow which is proportional to the voltage difference across it, then we define its resistance as the constant ratio
The units of resistance are volts/ampere, usually abbreviated as ohms, symbolized with the capital Greek letter omega, \(\Omega\).

**Example 7: Resistance of a lightbulb**

A flashlight bulb powered by a 9-volt battery has a resistance of 10 \(\Omega\). How much current will it draw?

Solving the definition of resistance for \(I\), we find

\[
I = \frac{\Delta V}{R} = \frac{0.9\text{ V}}{10\Omega} = 0.09\text{ A}
\]

Ohm's law states that many substances, including many solids and some liquids, display this kind of behavior, at least for voltages that are not too large. The fact that Ohm's law is called a “law” should not be taken to mean that all materials obey it, or that it has the same fundamental importance as Newton's laws, for example. Materials are called **ohmic** or **nonohmic**, depending on whether they obey Ohm's law.

If objects of the same size and shape made from two different ohmic materials have different resistances, we can say that one material is more resistive than the other, or equivalently that it is less conductive. Materials, such as metals, that are very conductive are said to be good **conductors**. Those that are extremely poor conductors, for example wood or rubber, are classified as **insulators**. There is no sharp distinction between the two classes of materials. Some, such as silicon, lie midway between the two extremes, and are called semiconductors.
On an intuitive level, we can understand the idea of resistance by making the sounds “hhhhhh” and “fffff.” To make air flow out of your mouth, you use your diaphragm to compress the air in your chest. The pressure difference between your chest and the air outside your mouth is analogous to a voltage difference. When you make the “h” sound, you form your mouth and throat in a way that allows air to flow easily. The large flow of air is like a large current. Dividing by a large current in the definition of resistance means that we get a small resistance. We say that the small resistance of your mouth and throat allows a large current to flow. When you make the “f” sound, you increase the resistance and cause a smaller current to flow.

![Diagram of four objects](image)

f / Four objects made of the same substance have different resistances.

Note that although the resistance of an object depends on the substance it is made of, we cannot speak simply of the “resistance of gold” or the “resistance of wood.” Figure f shows four examples of objects that have had wires attached at the ends as electrical connections. If they were made of the same substance, they would all nevertheless have different resistances because of their different sizes and shapes. A more detailed discussion will be more natural in the context of the following chapter, but it should not be too surprising that the resistance of f/2 will be greater than that of f/1 --- the image of water flowing through a pipe, however incorrect, gives us the right intuition. Object f/3 will have a smaller resistance than f/1 because the charged particles have less of it to get through.

---

**Superconductors**

All materials display some variation in resistance according to temperature (a fact that is used in thermostats to make a thermometer that can be easily interfaced to an electric circuit). More spectacularly, most metals have been found to exhibit a sudden change to zero resistance when cooled to a certain critical temperature. They are then said to be superconductors. Theoretically, superconductors should make a great many exciting devices possible, for example coiled-wire magnets that could be used to levitate trains. In practice, the critical temperatures of all metals are very low, and the resulting need for extreme refrigeration has made their use uneconomical except for such specialized applications as particle accelerators for physics research.
A superconducting segment of the ATLAS accelerator at Argonne National Laboratory near Chicago. It is used to accelerate beams of ions to a few percent of the speed of light for nuclear physics research. The shiny silver-colored surfaces are made of the element niobium, which is a superconductor at relatively high temperatures compared to other metals --- relatively high meaning the temperature of liquid helium! The beam of ions passes through the holes in the two small cylinders on the ends of the curved rods. Charge is shuffled back and forth between them at a frequency of 12 million cycles per second, so that they take turns being positive and negative. The positively charged beam consists of short spurts, each timed so that when it is in one of the segments it will be pulled forward by negative charge on the cylinder in front of it and pushed forward by the positively charged one behind. The huge currents involved would quickly melt any metal that was not superconducting, but in a superconductor they produce no heat at all.

But scientists have recently made the surprising discovery that certain ceramics are superconductors at less extreme temperatures. The technological barrier is now in finding practical methods for making wire out of these brittle materials. Wall Street is currently investing billions of dollars in developing superconducting devices for cellular phone relay stations based on these materials. In 2001, the city of Copenhagen replaced a short section of its electrical power trunks with superconducting cables, and they are now in operation and supplying power to customers.

There is currently no satisfactory theory of superconductivity in general, although superconductivity in metals is understood fairly well. Unfortunately I have yet to find a fundamental explanation of superconductivity in metals that works at the introductory level.

**Example 8: Finding charge given current**

In the segment of the ATLAS accelerator shown in figure g, the current flowing back and forth between the two cylinders is given by \(I = a \cos bt\). What is the charge on one of the cylinders as a function of time?
We are given the current and want to find the charge, i.e., we are given the derivative and we want to find
the original function that would give that derivative. This means we need to integrate:
\[
\begin{align*}
q &= \int I \, dt \\
&= \int a \cos bt \, dt \\
&= \frac{a}{b}\sin bt + q_{o},
\end{align*}
\]
where \(q_{o}\) is a constant of integration.

We can interpret this in order to explain why a superconductor needs to be used. The constant \(b\) must be very large,
since the current is supposed to oscillate back and forth millions of times a second. Looking at the final result, we see that
if \(b\) is a very large number, and \(q\) is to be a significant amount of charge, then \(a\) must be a very large number as
well. If \(a\) is numerically large, then the current must be very large, so it would heat the accelerator too much if it was
flowing through an ordinary conductor.

---

**Constant voltage throughout a conductor**

The idea of a superconductor leads us to the question of how we should expect an object to behave if it is made of a very good
conductor. Superconductors are an extreme case, but often a metal wire can be thought of as a perfect conductor, for example
if the parts of the circuit other than the wire are made of much less conductive materials. What happens if \(R\) equals zero in
the equation \(R = \Delta V/I\)? The result of dividing two numbers can only be zero if the number on top equals zero. This tells
us that if we pick any two points in a perfect conductor, the voltage difference between them must be zero. In other words, the
entire conductor must be at the same voltage.

---

h / 1. The finger deposits charges on the solid, spherical, metal doorknob and is then withdrawn. 2. Almost instantaneously,
the charges’ mutual repulsion makes them redistribute themselves uniformly on the surface of the sphere. The only excess
charge is on the surface; charges do exist in the atoms that form the interior of the sphere, but they are balanced. Charges on
the interior feel zero total electrical force from the ones at the surface. Charges at the surface experience a net outward
repulsion, but this is canceled out by the force that keep them from escaping into the air. 3. A voltmeter shows zero difference
in voltage between any two points on the interior or surface of the sphere. If the voltage difference wasn't zero, then energy
could be released by the flow of charge from one point to the other; this only happens before equilibrium is reached.

Constant voltage means that no work would be done on a charge as it moved from one point in the conductor to another. If
zero work was done only along a certain path between two specific points, it might mean that positive work was done along
part of the path and negative work along the rest, resulting in a cancellation. But there is no way that the work could come out
to be zero for all possible paths unless the electrical force on a charge was in fact zero at every point. Suppose, for example,
that you build up a static charge by scuffing your feet on a carpet, and then you deposit some of that charge onto a doorknob,
which is a good conductor. How can all that charge be in the doorknob without creating any electrical force at any point inside it? The only possible answer is that the charge moves around until it has spread itself into just the right configuration so that the forces exerted by all the little bits of excess surface charge on any charged particle within the doorknob exactly cancel out.

We can explain this behavior if we assume that the charge placed on the doorknob eventually settles down into a stable equilibrium. Since the doorknob is a conductor, the charge is free to move through it. If it was free to move and any part of it did experience a nonzero total force from the rest of the charge, then it would move, and we would not have an equilibrium.

Excess charge placed on a conductor, once it reaches its equilibrium configuration, is entirely on the surface, not on the interior. This should be intuitively reasonable in figure h, for example, since the charges are all repelling each other. A proof is given in example 35 on p. 623.

Since wires are good conductors, constancy of voltage throughout a conductor provides a convenient freedom in hooking up a voltmeter to a circuit. In figure i, points B and C are on the same piece of conducting wire, so \(V_B = V_C\). Measuring \(V_B - V_A\) gives the same result as measuring \(V_C - V_A\).

**Example 9: The lightning rod**

Suppose you have a pear-shaped conductor like the one in figure j/1. Since the pear is a conductor, there are free charges everywhere inside it. Panels 1 and 2 of the figure show a computer simulation with 100 identical electric charges. In 1, the charges are released at random positions inside the pear. Repulsion causes them all to fly outward onto the surface and then settle down into an orderly but nonuniform pattern.
In 1 and 2, charges that are visible on the front surface of the conductor are shown as solid dots; the others would have to be seen through the conductor, which we imagine is semi-transparent.

We might not have been able to guess the pattern in advance, but we can verify that some of its features make sense. For example, charge A has more neighbors on the right than on the left, which would tend to make it accelerate off to the left. But when we look at the picture as a whole, it appears reasonable that this is prevented by the larger number of more distant charges on its left than on its right.

There also seems to be a pattern to the nonuniformity: the charges collect more densely in areas like B, where the surface is strongly curved, and less densely in flatter areas like C.

To understand the reason for this pattern, consider j/3. Two conducting spheres are connected by a conducting wire. Since the whole apparatus is conducting, it must all be at one voltage. As shown in problem 37 on p. 550, the density of charge is greater on the smaller sphere. This is an example of a more general fact observed in j/2, which is that the charge on a conductor packs itself more densely in areas that are more sharply curved.
Similar reasoning shows why Benjamin Franklin used a sharp tip when he invented the lightning rod. The charged stormclouds induce positive and negative charges to move to opposite ends of the rod. At the pointed upper end of the rod, the charge tends to concentrate at the point, and this charge attracts the lightning. The same effect can sometimes be seen when a scrap of aluminum foil is inadvertently put in a microwave oven. Modern experiments (Moore et al., Journal of Applied Meteorology 39 (1999) 593) show that although a sharp tip is best at starting a spark, a more moderate curve, like the right-hand tip of the pear in this example, is better at successfully sustaining the spark for long enough to connect a discharge to the clouds.

**Short circuits**

So far we have been assuming a perfect conductor. What if it is a good conductor, but not a perfect one? Then we can solve for \( \Delta V = IR \). An ordinary-sized current will make a very small result when we multiply it by the resistance of a good conductor such as a metal wire. The voltage throughout the wire will then be nearly constant. If, on the other hand, the current is extremely large, we can have a significant voltage difference. This is what happens in a short-circuit: a circuit in which a low-resistance pathway connects the two sides of a voltage source. Note that this is much more specific than the popular use of the term to indicate any electrical malfunction at all. If, for example, you short-circuit a 9-volt battery as shown in figure k, you will produce perhaps a thousand amperes of current, leading to a very large value of \( P = \Delta V \). The wire gets hot!

![Short-circuiting a battery](image)

k / Short-circuiting a battery. Warning: you can burn yourself this way or start a fire! If you want to try this, try making the connection only very briefly, use a low-voltage battery, and avoid touching the battery or the wire, both of which will get hot.

**self-check:**

What would happen to the battery in this kind of short circuit?

(Answer in the back of the PDF version of the book)

At this stage, most students have a hard time understanding why resistors would be used inside a radio or a computer. We obviously want a lightbulb or an electric stove to have a circuit element that resists the flow of electricity and heats up, but heating is undesirable in radios and computers. Without going too far afield, let's use a mechanical analogy to get a general idea of why a resistor would be used in a radio.
The main parts of a radio receiver are an antenna, a tuner for selecting the frequency, and an amplifier to strengthen the signal sufficiently to drive a speaker. The tuner resonates at the selected frequency, just as in the examples of mechanical resonance discussed in 3. The behavior of a mechanical resonator depends on three things: its inertia, its stiffness, and the amount of friction or damping. The first two parameters locate the peak of the resonance curve, while the damping determines the width of the resonance. In the radio tuner we have an electrically vibrating system that resonates at a particular frequency. Instead of a physical object moving back and forth, these vibrations consist of electrical currents that flow first in one direction and then in the other. In a mechanical system, damping means taking energy out of the vibration in the form of heat, and exactly the same idea applies to an electrical system: the resistor supplies the damping, and therefore controls the width of the resonance. If we set out to eliminate all resistance in the tuner circuit, by not building in a resistor and by somehow getting rid of all the inherent electrical resistance of the wires, we would have a useless radio. The tuner's resonance would be so narrow that we could never get close enough to the right frequency to bring in the station. The roles of inertia and stiffness are played by other circuit elements we have not discusses (a capacitor and a coil).

### Resistors

Inside any electronic gadget you will see quite a few little circuit elements like the one shown below. These resistors are simply a cylinder of ohmic material with wires attached to the end.

<table>
<thead>
<tr>
<th>color</th>
<th>meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>black</td>
<td>0</td>
</tr>
<tr>
<td>brown</td>
<td>1</td>
</tr>
<tr>
<td>red</td>
<td>2</td>
</tr>
<tr>
<td>orange</td>
<td>3</td>
</tr>
<tr>
<td>yellow</td>
<td>4</td>
</tr>
<tr>
<td>green</td>
<td>5</td>
</tr>
<tr>
<td>blue</td>
<td>6</td>
</tr>
<tr>
<td>violet</td>
<td>7</td>
</tr>
<tr>
<td>gray</td>
<td>8</td>
</tr>
<tr>
<td>white</td>
<td>9</td>
</tr>
<tr>
<td>silver</td>
<td>±10%</td>
</tr>
<tr>
<td>gold</td>
<td>±5%</td>
</tr>
</tbody>
</table>
Many electrical devices are based on electrical resistance and Ohm's law, even if they do not have little components in them that look like the usual resistor. The following are some examples.

**Lightbulb**

There is nothing special about a lightbulb filament --- you can easily make a lightbulb by cutting a narrow waist into a metallic gum wrapper and connecting the wrapper across the terminals of a 9-volt battery. The trouble is that it will instantly burn out. Edison solved this technical challenge by encasing the filament in an evacuated bulb, which prevented burning, since burning requires oxygen.

**Polygraph**

The polygraph, or “lie detector,” is really just a set of meters for recording physical measures of the subject's psychological stress, such as sweating and quickened heartbeat. The real-time sweat measurement works on the principle that dry skin is a good insulator, but sweaty skin is a conductor. Of course a truthful subject may become nervous simply because of the situation, and a practiced liar may not even break a sweat. The method's practitioners claim that they can tell the difference, but you should think twice before allowing yourself to be polygraph tested. Most U.S. courts exclude all polygraph evidence, but some employers attempt to screen out dishonest employees by polygraph testing job applicants, an abuse that ranks with such pseudoscience as handwriting analysis.
**Fuse**

A fuse is a device inserted in a circuit tollbooth-style in the same manner as an ammeter. It is simply a piece of wire made of metals having a relatively low melting point. If too much current passes through the fuse, it melts, opening the circuit. The purpose is to make sure that the building's wires do not carry so much current that they themselves will get hot enough to start a fire. Most modern houses use circuit breakers instead of fuses, although fuses are still common in cars and small devices. A circuit breaker is a switch operated by a coiled-wire magnet, which opens the circuit when enough current flows. The advantage is that once you turn off some of the appliances that were sucking up too much current, you can immediately flip the switch closed. In the days of fuses, one might get caught without a replacement fuse, or even be tempted to stuff aluminum foil in as a replacement, defeating the safety feature.

**Voltmeter**

A voltmeter is nothing more than an ammeter with an additional high-value resistor through which the current is also forced to flow. Ohm's law relates the current through the resistor is related directly to the voltage difference across it, so the meter can be calibrated in units of volts based on the known value of the resistor. The voltmeter's two probes are touched to the two locations in a circuit between which we wish to measure the voltage difference, o/2. Note how cumbersome this type of drawing is, and how difficult it can be to tell what is connected to what. This is why electrical drawing are usually shown in schematic form. Figure o/3 is a schematic representation of figure o/2.

The setups for measuring current and voltage are different. When we are measuring current, we are finding “how much stuff goes through,” so we place the ammeter where all the current is forced to go through it. Voltage, however, is not “stuff that goes through,” it is a measure of electrical energy. If an ammeter is like the meter that measures your water use, a voltmeter is like a measuring stick that tells you how high a waterfall is, so that you can determine how much energy will be released by each kilogram of falling water. We do not want to force the water to go through the measuring stick! The arrangement in figure o/3 is a (parallel) circuit: one in there are “forks in the road” where some of the current will flow one way and some will flow the other. Figure o/4 is said to be wired in series: all the current will visit all the circuit elements one after the other. We will deal with series and parallel circuits in more detail in the following chapter.

If you inserted a voltmeter incorrectly, in series with the bulb and battery, its large internal resistance would cut the current down so low that the bulb would go out. You would have severely disturbed the behavior of the circuit by trying to measure something about it.
Incorrectly placing an ammeter in parallel is likely to be even more disconcerting. The ammeter has nothing but wire inside it to provide resistance, so given the choice, most of the current will flow through it rather than through the bulb. So much current will flow through the ammeter, in fact, that there is a danger of burning out the battery or the meter or both! For this reason, most ammeters have fuses or circuit breakers inside. Some models will trip their circuit breakers and make an audible alarm in this situation, while others will simply blow a fuse and stop working until you replace it.

**Discussion Questions**

◊ In figure o/1, would it make any difference in the voltage measurement if we touched the voltmeter's probes to different points along the same segments of wire?

◊ Explain why it would be incorrect to define resistance as the amount of charge the resistor allows to flow.

### 9.1.5 Current-conducting properties of materials

Ohm's law has a remarkable property, which is that current will flow even in response to a voltage difference that is as small as we care to make it. In the analogy of pushing a crate across a floor, it is as though even a flea could slide the crate across the floor, albeit at some very low speed. The flea cannot do this because of static friction, which we can think of as an effect arising from the tendency of the microscopic bumps and valleys in the crate and floor to lock together. The fact that Ohm's law holds for nearly all solids has an interesting interpretation: at least some of the electrons are not “locked down” at all to any specific atom.

More generally we can ask how charge actually flows in various solids, liquids, and gases. This will lead us to the explanations of many interesting phenomena, including lightning, the bluish crust that builds up on the terminals of car batteries, and the need for electrolytes in sports drinks.

### Solids

In atomic terms, the defining characteristic of a solid is that its atoms are packed together, and the nuclei cannot move very far from their equilibrium positions. It makes sense, then, that electrons, not ions, would be the charge carriers when currents flow in solids. This fact was established experimentally by Tolman and Stewart, in an experiment in which they spun a large coil of wire and then abruptly stopped it. They observed a current in the wire immediately after the coil was stopped, which indicated that charged particles that were not permanently locked to a specific atom had continued to move because of their own inertia, even after the material of the wire in general stopped. The direction of the current showed that it was negatively charged particles that kept moving. The current only lasted for an instant, however; as the negatively charged particles collected at the downstream end of the wire, farther particles were prevented joining them due to their electrical repulsion, as well as the attraction from the upstream end, which was left with a net positive charge. Tolman and Stewart were even able to determine the mass-to-charge ratio of the particles. We need not go into the details of the analysis here, but particles with high mass would be difficult to decelerate, leading to a stronger and longer pulse of current, while particles with high charge would feel stronger electrical forces decelerating them, which would cause a weaker and shorter pulse. The mass-to-charge ratio thus
determined was consistent with the \(m/q\) of the electron to within the accuracy of the experiment, which essentially established that the particles were electrons.

The fact that only electrons carry current in solids, not ions, has many important implications. For one thing, it explains why wires don't fray or turn to dust after carrying current for a long time. Electrons are very small (perhaps even pointlike), and it is easy to imagine them passing between the cracks among the atoms without creating holes or fractures in the atomic framework. For those who know a little chemistry, it also explains why all the best conductors are on the left side of the periodic table. The elements in that area are the ones that have only a very loose hold on their outermost electrons.

---

**Gases**

The molecules in a gas spend most of their time separated from each other by significant distances, so it is not possible for them to conduct electricity the way solids do, by handing off electrons from atom to atom. It is therefore not surprising that gases are good insulators.

Gases are also usually nonohmic. As opposite charges build up on a stormcloud and the ground below, the voltage difference becomes greater and greater. Zero current flows, however, until finally the voltage reaches a certain threshold and we have an impressive example of what is known as a spark or electrical discharge. If air was ohmic, the current between the cloud and the ground would simply increase steadily as the voltage difference increased, rather than being zero until a threshold was reached. This behavior can be explained as follows. At some point, the electrical forces on the air electrons and nuclei of the air molecules become so strong that electrons are ripped right off of some of the molecules. The electrons then accelerate toward either the cloud or the ground, whichever is positively charged, and the positive ions accelerate the opposite way. As these charge carriers accelerate, they strike and ionize other molecules, which produces a rapidly growing cascade.

---

**Liquids**

Molecules in a liquid are able to slide past each other, so ions as well as electrons can carry currents. Pure water is a poor conductor because the water molecules tend to hold onto their electrons strongly, and there are therefore not many electrons or ions available to move. Water can become quite a good conductor, however, with the addition of even a small amount of certain substances called electrolytes, which are typically salts. For example, if we add table salt, NaCl, to water, the NaCl molecules dissolve into \(Na^+(\ ^{\text{\text{\dagger}}})\) and \(Cl^{-}(\ ^{\text{\text{\dagger}}})\) ions, which can then move and create currents. This is why electric currents can flow among the cells in our bodies: cellular fluid is quite salty. When we sweat, we lose not just water but electrolytes, so dehydration plays havoc with our cells' electrical systems. It is for this reason that electrolytes are included in sports drinks and formulas for rehydrating infants who have diarrhea.

Since current flow in liquids involves entire ions, it is not surprising that we can see physical evidence when it has occurred. For example, after a car battery has been in use for a while, the \(\text{H}_2\text{SO}_4\) battery acid becomes depleted of hydrogen ions, which are the main charge carriers that complete the circuit on the inside of the battery. The leftover \(\text{SO}_4\) then forms a visible blue crust on the battery posts.
Speed of currents and electrical signals

When I talk on the phone to my mother in law two thousand miles away, I do not notice any delay while the signal makes its way back and forth. Electrical signals therefore must travel very quickly, but how fast exactly? The answer is rather subtle. For the sake of concreteness, let's restrict ourselves to currents in metals, which consist of electrons.

The electrons themselves are only moving at speeds of perhaps a few thousand miles per hour, and their motion is mostly random thermal motion. This shows that the electrons in my phone cannot possibly be zipping back and forth between California and New York fast enough to carry the signals. Even if their thousand-mile-an-hour motion was organized rather than random, it would still take them many minutes to get there. Realistically, it will take the average electron even longer than that to make the trip. The current in the wire consists only of a slow overall drift, at a speed on the order of a few centimeters per second, superimposed on the more rapid random motion. We can compare this with the slow westward drift in the population of the U.S. If we could make a movie of the motion of all the people in the U.S. from outer space, and could watch it at high speed so that the people appeared to be scurrying around like ants, we would think that the motion was fairly random, and we would not immediately notice the westward drift. Only after many years would we realize that the number of people heading west over the Sierras had exceeded the number going east, so that California increased its share of the country's population.

So why are electrical signals so fast if the average drift speed of electrons is so slow? The answer is that a disturbance in an electrical system can move much more quickly than the charges themselves. It is as though we filled a pipe with golf balls and then inserted an extra ball at one end, causing a ball to fall out at the other end. The force propagated to the other end in a fraction of a second, but the balls themselves only traveled a few centimeters in that time.

Because the reality of current conduction is so complex, we often describe things using mental shortcuts that are technically incorrect. This is OK as long as we know that they are just shortcuts. For example, suppose the presidents of France and Russia shake hands, and the French politician has inadvertently picked up a positive electrical charge, which shocks the Russian. We may say that the excess positively charged particles in the French leader's body, which all repel each other, take the handshake as an opportunity to get farther apart by spreading out into two bodies rather than one. In reality, it would be a matter of minutes before the ions in one person's body could actually drift deep into the other's. What really happens is that throughout the body of the recipient of the shock there are already various positive and negative ions which are free to move. Even before the perpetrator's charged hand touches the victim's sweaty palm, the charges in the shocker's body begin to repel the positive ions and attract the negative ions in the other person. The split-second sensation of shock is caused by the sudden jumping of the victim's ions by distances of perhaps a micrometer, this effect occurring simultaneously throughout the whole body, although more violently in the hand and arm, which are closer to the other person.

Contributors

Benjamin Crowell (Fullerton College). Conceptual Physics is copyrighted with a CC-BY-SA license.