7.4: Fission and Fusion

Fission

Find \( Q \) for the fission reaction:

\[
\ce{ ^{1}_0n + ^{235}_{92}U \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_0n } \tag{fission1}
\]

As we noticed in our discussion of binding energy, nuclei much more massive than iron actually have less binding energy per nucleon than iron. Thus, if some method existed by which these massive nuclei could be “broken up” into iron-sized fragments energy would be released. This method is termed nuclear fission.

Although some nuclei will undergo spontaneous fission, most fission reactions of interest involve induced fission, in which an incoming projectile, typically a neutron, collides with the target nuclei and initiates the fission process. A typical example is

\[
\ce{ n + _{92} ^{235}U \Rightarrow _{56}^{141}Ba + _{36}^{92}Kr + 3n}
\]

The \( Q \) for this reaction is:

\[
Q = m_{U, \text{atomic}}c^2 - m_{Bd, \text{atomic}}c^2 - m_{Kr, \text{atomic}}c^2 - 2(m_{nc}c^2)
\]

\[
Q = (m_{U, \text{atomic}} - m_{Bd, \text{atomic}} - m_{Kr, \text{atomic}} - 2m_{nc})c^2
\]
Thus, for every $^{235}\text{U}$ nucleus that undergoes fission by this process, 173.3 MeV is released.

It should be pointed out that there is nothing special about this particular fission reaction. When struck by the incoming neutron, the uranium nucleus can fission in many, many different ways, with the vast majority of these fission processes releasing between 150 and 200 MeV.

Additionally, the fission fragments ($^{141}\text{Ba}$ and $^{92}\text{Kr}$) are themselves radioactive. Their subsequent decay(s) into a stable form will add to the total energy released by the fission process.

Finally, special mention should be made of the neutrons released by the fission reaction. These neutrons can be used to fission additional uranium nuclei, leading to more energy and still more neutrons. This chain reaction is crucial to the construction of a fission weapon, and crucial to control in a fission power plant.

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Fusion

Find $\langle Q \rangle$ for the fusion reaction:

$$\langle \text{ } ^2_1\text{H} + ^2_1\text{H} \rightarrow ^3_2\text{He} + \gamma \rangle$$

Again, as we noticed in our discussion of binding energy, nuclei much less massive than iron actually have less binding energy per nucleon than iron. Thus, if some method existed by which these very low mass nuclei could be “squeezed together” into larger nuclei energy would be released. This method is termed nuclear fusion.

There are no examples of spontaneous fusion. Since all nuclei are positively charged, the electromagnetic repulsion between two nuclei makes getting them close enough together to “fall” into each other’s potential wells very difficult. In fact, quantum mechanical tunneling is crucial to understanding the fusion that takes place in the sun. Without the ability of two nuclei to tunnel through the relatively large electrostatic barrier separating them, the sun would not shine!

The $\langle Q \rangle$ for the fusion reaction given is:

$$\langle Q \rangle = (m_{2\text{H, atomic}}c^2 - m_e c^2) + (m_{\text{H, atomic}}c^2 - m_e c^2) - (m_{3\text{He, atomic}}c^2 - 2m_e c^2)$$

$$\langle Q \rangle = (2.014102 + 1.007825 - 3.016029)c^2$$

$$\langle Q \rangle = 5.49 \text{ MeV}$$

Thus, 5.49 MeV is released for every occurrence of this reaction.

This result can be compared to the fission result a number of different ways. First, although less energy is released per reaction by this fusion reaction, more energy is released per kg of reactants. For example, dividing the $\langle Q \rangle$ by the atomic mass of the
reactants yields:

\[
\frac{Q}{A_{\text{fission}}} = \frac{173.3 \text{ MeV}}{236} = 0.7 \text{ MeV}
\]

\[
\frac{Q}{A_{\text{fission}}} = \frac{5.49 \text{ MeV}}{3} = 1.8 \text{ MeV}
\]

Additionally, not only is the input fuel for the fusion reaction incredibly common (there’s a bit more water on the earth than there is \(^{235}\text{U}\)), the output materials have a relatively low level of radioactivity in comparison to the very nasty fragments created by fission.

There is, however, one huge downside to fusion; the energies and pressures needed to control fusion are still beyond our technological abilities! We can build fusion bombs, but not fusion power plants.